The Two Sides of Envy*

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Abstract

The two sides of envy, destructive and constructive, give rise to qualitatively different equilibria, depending on the economic, institutional, and cultural environment. If investment opportunities are scarce, inequality is high, property rights are not secure, and social comparisons are strong, society is likely to be in the "fear equilibrium," in which better endowed agents underinvest in order to avoid destructive envy of the relatively poor. Otherwise, the standard "keeping up with the Joneses" competition arises, and envy is satisfied through suboptimally high efforts. Economic growth expands the production possibilities frontier and triggers an endogenous transition from one equilibrium to the other causing a qualitative shift in the relationship between envy and economic performance: envy-avoidance behavior with its adverse effect on investment paves the way to creative emulation. From a welfare perspective, better institutions and wealth redistribution that move the society away from the low-output fear equilibrium need not be Pareto improving in the short run, as they unleash the negative consumption externality. In the long run, such policies contribute to an increase in social welfare due to enhanced productivity growth.

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Destructive egalitarian envy dictates the darkest pages of history; hierarchical and creative emulation narrates its splendor.

Gonzalo Fernández de la Mora (1987)
Egalitarian Envy: The Political Foundations of Social Justice

1 Introduction

The interplay between culture and economic activity has recently become the centerpiece of a vibrant interdisciplinary research agenda.[1] Trust, religion, family ties, and risk attitudes are among the attributes argued to have profound effects on economic outcomes.[2] Although concern for relative standing, envy for short, is widely recognized as a salient feature of individuals interacting in a society, there is no agreement on how it affects the economy, either directly, through its impact on incentives to work and invest, or indirectly, through its connection to institutions and social norms.[3] In some parts of the world people engage in conspicuous consumption and overwork, driven by competition for status, while in others they hide their wealth and underinvest, constrained by the fear of malicious envy.

This paper develops a unified framework capturing qualitatively different equilibria that emerge in the presence of envy, depending on the economic, institutional, and cultural environment. It sheds new light on the implications of envy for economic performance and social welfare and examines its changing role in the process of development.

Throughout the paper envy is taken to be a characteristic of preferences that makes people care about how their own consumption level compares to that of their reference group. This operational definition implies that envy can be satisfied in two major ways: by increasing own outcome (constructive envy) and by decreasing the outcome of the reference

[1] The term "culture" refers to features of preferences, beliefs, and social norms (Fernández, 2011).

[2] For a state-of-the-art overview see various chapters in Benhabib et al. (2011). A different strand of literature looks at the endogenous formation of preferences in the context of long-run economic development, with recent examples including Doepke and Zilibotti (2008) on patience and work ethic and Galor and Michalopoulos (2012) on risk aversion.

[3] A number of evolutionary theoretic explanations have been proposed for why people care about relative standing, see Hopkins (2008, section 3) and Robson and Samuelson (2011, section 4.2). Evidence documenting people's concern for relative standing is abundant and comes from empirical happiness research (Luttmer, 2005), job satisfaction studies (Card et al., 2012), experimental economics (Zizzo, 2003; Rustichini, 2008), neuroscience (Fliessbach et al., 2007), and surveys (Solnick and Hemenway, 2005; Clark and Senik, 2010); see Clark et al. (2008, section 3) and Frank and Heffetz (2011, section 3) for an overview.

group (destructive envy).[4] Given the two sides of envy, individuals face a fundamental trade-off. On the one hand, they strive for higher relative standing. On the other hand, they want to avoid the destructive envy of those falling behind. The way this trade-off is resolved shapes the role of envy in society.

The basic theory is set up as a simple two-stage dynamic "envy game" between two social groups or individuals who are each other's reference points. In the first stage of the game each individual may choose to allocate part of his time to productive investment that, combined with initial endowment, raises his future productivity. In the second stage the available time is split between own production and the disruption of the other individual's production process. The optimal allocation of time between productive and destructive activities in this setup depends on the scope of available investment opportunities, disparity of investment outcomes, and tolerance for inequality, the latter endogenously determined by the level of property rights protection (effectiveness of destructive technology) and the strength of social comparisons. The unique equilibrium of the game belongs to one of three qualitatively different classes whose features are broadly consistent with the conflicting evidence on the role of envy across societies.

If the initial inequality is low, tolerance for inequality is high, and peaceful investment opportunities are abundant, the familiar "keeping up with the Joneses" (KUJ) equilibrium arises. In this case individuals compete peacefully for their relative standing, and the consumption externality leads to high effort and output. These are typical characteristics of a consumer society prominently documented for the U.S. by Schor (1991) and Matt (2003).

Higher inequality, lower tolerance for inequality, and scarce investment opportunities lead to the "fear equilibrium," in which the better endowed individual anticipates destructive envy and prevents it by restricting his effort. Such envy-avoidance behavior is typical for traditional agricultural communities in developing economies, where the fear of inciting envy discourages production (Foster, 1979; Dow, 1981). It is also characteristic for the emerging markets, where potential entrepreneurs are reluctant to start new business that can provoke envious retaliation of others (Mui, 1995).

[4] An alternative option is to drop out of competition for status (Banerjee, 1990; Barnett et al., 2010). Yet another possibility is to redefine the reference group, see Falk and Knell (2004) for a model with endogenous formation of reference standards.

Finally, if the distribution of endowments is highly unequal, and tolerance for inequality is very low, as is investment productivity, a "destructive equilibrium" arises, in which actual conflict takes place and time is wasted to satisfy envy.[5]

The qualitatively different nature of these equilibria leads to opposite effects of envy on aggregate economic performance. In the KUJ equilibrium stronger envy increases effort and output by intensifying status competition, while in the fear equilibrium it reinforces the envy-avoidance behavior further discouraging investment.

To explore the endogenous evolution of the role of envy in the process of economic development, the basic model is embedded in an endogenous growth framework, in which productivity rises due to learning-by-doing and knowledge spillovers. Rising productivity expands investment opportunities and the society experiences an endogenous transition from the fear of envy to the KUJ equilibrium: envy-avoidance behavior, dictated by the destructive side of envy, eventually paves the way to emulation, driven by its constructive side. Over the course of this fundamental transition envy feeds back into productivity growth by constraining or spurring investment, depending on the type of equilibrium.

In the dynamic setting, inequality and tolerance parameters not only affect economic outcomes at a given point in time but also determine the division of the transition process into alternative "envy regimes" and the overall long-run trajectory of the economy. The transition from fear to competition can be delayed or accelerated by various factors affecting the strength of social comparisons and the relative attractiveness of productive and destructive effort, such as religion, political ideology, institutions, and exogenous productivity spillovers.

The striking difference between alternative equilibria is also reflected in a comparative welfare analysis. While better institutions and wealth redistribution can move the society from the low-output fear equilibrium to the high-output KUJ equilibrium, such change need not be welfare enhancing if it triggers the "rat race" competition. That is, individuals may be happier in the fear equilibrium, since the threat of destructive envy constrains the suboptimally high efforts that would have been induced in the KUJ equilibrium. However, in the long-run, from the viewpoint of future generations, such policies that shift the society to a KUJ trajectory contribute to improving social welfare due to additional productivity growth.

[5] Allowing for transfers replaces destructive activities of the poor with "voluntary" sharing of the rich, reflecting the evidence on the fear-of-envy-motivated redistribution in peasant societies of Latin America (Cancian, 1965), Southeast Asia (Scott, 1976), and Africa (Platteau, 2000). This possibility is examined in Gershman (2012).

The rest of the paper is organized as follows. Section 2 summarizes the evidence on the two sides of envy. Section 3 lays out the basic model and examines comparative statics. Section 4 looks at the long-run dynamics. Section 5 conducts the welfare analysis. Section 6 concludes. The Appendix contains the detailed versions of some key results. Supplementary materials available online include the detailed proofs of all statements and an extension of the model that incorporates bequest dynamics.

2 Evidence on the two sides of envy

The affluence of the rich excites the indignation of the poor, who are often both driven by want, and prompted by envy, to invade his possessions.

Adam Smith (1776)
The Wealth of Nations

As wealth increases, the continued stimulus of emulation would make each man strive to surpass, or at least not fall below, his neighbours, in this.

Richard Whately (1831)
Introductory Lectures on Political Economy

As exemplified by the quotations above, the distinction between constructive and destructive sides of envy goes back to classical economics. Since then, this dichotomy has been discussed extensively by anthropologists (Foster, 1972), philosophers (D'Arms and Kerr, 2008), political scientists (Fernández de la Mora, 1987), psychologists (Smith and Kim, 2007), sociologists (Schoeck, 1969; Clanton, 2006), and recently got a renewed interest from economists (Elster, 1991; Zizzo, 2008; Mitsopoulos, 2009).[6]

2.1 Destructive envy and the fear of it

Evidence on the destructive potential of envy comes primarily from the developing world. For instance, recent data from the Afrobarometer surveys indicate that envy is perceived as an important source of conflict.[7] Respondents were asked the following question: "Over what sort of problems do violent conflicts most often arise between different groups in this country?" They were offered to choose three most important problems from a list of several dozens of possible answers (such as religion, ethnic differences, economic issues). Remarkably, in nine countries, where the list featured "envy/gossip" as a potential source of violent conflict, an average of over 9% of respondents put it among the top three problems.

[6]Certain psychological approaches treat "benign" and "malicious" envy as two separate emotions (van de Ven et al., 2009). The present theory is crucially different in that the emotion, envy, is the same, but its manifestation is an equilibrium outcome.

[7]Second round, 2002–2003; raw data available at `http://www.afrobarometer.org`.

The threat of destructive envy naturally causes a rational fear of it which motivates envy-avoidance behavior. Numerous examples of such behavior come from anthropological research on small-scale agricultural communities around the world. According to Foster (1972), in peasant societies "a man fears being envied for what he has, and wishes to protect himself from the consequences of the envy of others" (p. 166). People are reluctant to exert effort or innovate since they expect sanctions in the form of plain destruction, forced redistribution, or envy-driven supernatural punishment.[8]

In Amatenango, a Mayan village in Mexico, "the fear of exciting envy acts as a brake on productive activity and the enjoyment of improved consumption resulting from wealth" (Nash, 1970, p. 95). Similarly, the "envious hostility of neighbors" discourages the villagers of Northern Sierra de Puebla, also in Mexico, to produce food beyond subsistence (Dow, 1981). In peasant communities of Southeast Asia people work "in large measure through the abrasive force of gossip and envy and the knowledge that the abandoned poor are likely to be a real and present danger to better-off villagers" (Scott, 1976, p. 5).

The fear of destructive envy is not an exclusive feature of simple agricultural societies. Mui (1995) focuses on the large industrial economies, Russia and China, in the process of transition to the free market. He brings up evidence on emerging cooperative restaurants and shops in the Soviet Union being regularly attacked by people resenting the success of those new businesses. Mui then tells a similar story of a Chinese peasant whose successful entrepreneurship provoked the envious neighbors to steal timber for his new house and kill his farm animals. "I dare not work too hard to get rich again" was his comment to the press. The fear of envy may be a particularly serious issue in societies with socialist experience marked by the ideology of material egalitarianism and neglect of private property rights, both of which reduce the tolerance for inequality.

2.2 Constructive envy and keeping up with the Joneses

A very different strand of evidence comes from developed economies, in which the constructive side of envy is predominant while the fear of destructive envy is virtually non-existent. As observed with dismay by the Christian Advocate newspaper in 1926, consumer societies obey a new version of the tenth commandment: "Thou shalt not be outdone by thy neighbor's house, thou shalt not be outdone by thy neighbor's wife, nor his manservant,

[8] The rational fear of envy becomes curiously embedded in cultural beliefs. The term "institutionalized envy" coined by Wolf (1955) summarizes the set of cultural control mechanisms related to the fear of envy including gossip, the fear of witchcraft, and the evil eye belief. See Gershman (2014) for more details.

nor his car, nor anything – irrespective of its price or thine own ability – anything that is thy neighbor's" (cited in Matt, 2003, p. 4).

Under "keeping up with the Joneses" envy acts as an additional incentive to work hard to be able to match the consumption pattern of the reference group. According to Schor (1991), the steady increase in work hours in the U.S. since the early 1970s is primarily due to the competitive effects of social comparisons. The positive association between concern for relative standing and labor supply in developed economies is also found in rigorous empirical research. Neumark and Postlewaite (1998) study the employment decisions of women using data from the U.S. National Longitudinal Survey of Youth and find evidence that those are partly driven by relative income (see also Park, 2010). Bowles and Park (2005) use aggregate data from ten OECD countries over the period 1963–1998 to show that greater earnings inequality is associated with longer work hours. They attribute this finding to the "Veblen effect" of the consumption of the rich on the less wealthy, that is, emulation. Pérez-Asenjo (2011) examines the data from the U.S. General Social Survey and reports that individuals work longer hours if their income falls relative to that of their respective reference groups.

Overworking caused by constructive envy and the welfare consequences of the KUJ-type competition have become the subject of recent research in the field of happiness economics (Graham, 2010). In particular, concern for relative standing is one of the keys to understanding why happiness and material well-being might not always go together. In the words of Schor (1991, p. 124), those caught in a race for relative standing "would be better off with more free time; but without cooperation, they will stick to the long hours, high consumption choice."[9]

2.3 From fear to competition: The case of Tzintzuntzan

As hinted in the previous discussion, the destructive side of envy and the fear of it dominate in developing communities with limited investment opportunities and poor institutional environment, while the constructive side of envy is prevalent in advanced consumer

[9] Some have argued that, in addition to its stimulating effect on labor supply, "wasteful" conspicuous consumption may hurt savings and, hence, economic growth (Frank, 2007). Cozzi (2004) argues instead that status competition can lead to increased capital accumulation. Corneo and Jeanne (1998) show that the effect of status competition on savings depends crucially on the assumption about the timing of "contests for status" over the life cycle. Moav and Neeman (2012) develop a model in which both human capital and conspicuous consumption serve as signals for income and show that in this setup concern for status can generate a poverty trap due to increasing marginal propensity to save.

economies. Thus, it is tempting to surmise that the role of envy evolves in the process of economic development. While such time-series evidence is difficult to obtain, one case study is particularly illuminating.

The Mexican village of Tzintzuntzan became famous all over the world due to the fieldwork of anthropologist George Foster who documented the life in this community for more than half a century. Part of his work represents an extensive case study on the evolution of envy-related behavior and culture over time.

During the first twenty years of Foster's fieldwork that started in 1944 Tzintzuntzan represented an isolated peasant community, marked by low productivity in agriculture, pottery, and a few other activities that shaped the economic basis of the village. One of the striking phenomena described by Foster is that villagers were reluctant to fully use even those limited opportunities that were available to them due the fear of envy-driven hostility of their neighbors. To pick one example, a relatively wealthy peasant refused to lay a cement or tile floor or cut windows in his rooms since he was "frankly afraid people will envy him" (Foster, 1979, p. 154). Such fear was a major obstacle to economic development and innovation exacerbating the vicious cycle between meager investment opportunities and destructive envy. Tzintzuntzan of those decades is not a unique case, but rather a typical representative of similar communities across the globe, as illustrated in section 2.1.

What is particularly interesting about Tzintzuntzan is the documented history of its remarkable transformation. As Foster describes in the epilogue of his book, the rise in economic opportunities came both from within the village and as a spillover from Mexico's impressive economic growth manifested in extensive infrastructure investment including new roads, electrification, expansion of health and educational facilities, and stimulation of tourism. As the villagers have become convinced of the reality of new opportunities, "they have shown ingenuity in responding to them" (Foster, 1979, p. 375).

These new opportunities deeply affected the envy-related behavior in the community: an increasing number of people seemed "no longer worried by the fear of negative sanctions if they spend conspicuously" (pp. 380–381). Generation by generation, the village was making a fundamental transition to a true consumer society: "Fearing envy, a few older people still hesitate to improve their houses, but most families routinely remodel old homes or build new ones, complete with gas stove, television set, expensive hi-fi radio-phonograph combinations, hot water shower baths, and other products of a commercial age" (p. 383).[10]

[10]This is in line with evidence cited by Moav and Neeman (2012) and others that even in poor economies some people engage in conspicuous consumption. The theory that follows identifies the conditions that facilitate a transition from the fear of envy to peaceful status competition in developing societies.

The transformation in the material life of the society brought about a deep change in the envy-related culture. Beliefs in the destructive force of envy, the "zero-sum thinking," and cognitive orientation that Foster dubbed the "image of limited good" eventually paved the way to consumer culture with open competition in "dress, hair styles, in weddings and baptisms, and in 'knowing city ways' " (p. 383).

It would be misleading to view such evolution of the role of envy as an idiosyncratic feature of small-scale peasant communities on their path to integration into larger economies. Elements of this fundamental transition are observed at different times in various societies. According to Matt (2003), the American society experienced an impressive transition of similar sort towards the beginning of the twentieth century: "Envy, which thirty years earlier had been considered a grave sin, was now regarded as a beneficial force for social progress and individual advancement" (p. 2). She attributes such transformation in values and behavior to the growing abundance of goods due to mass production which channeled envy into emulation rather than conflict, hostility, and strife.[11]

The controversial evidence on the role of envy in society poses a number of questions. How does the same feature, concern for relative standing, give rise to these qualitatively different cases? Under what conditions do the fear of envy or the KUJ competition emerge? What are the implications of social comparisons for economic performance and welfare? How do they change in the process of development? The first step towards thinking about these issues is to construct a unified framework reconciling the above evidence.

3 The basic model

3.1 Environment

Consider an economy populated by a sequence of non-overlapping generations, indexed by $t \geqslant 0$. Time is discrete, each generation lives for one period and consists of two equal-sized homogeneous groups of people. They differ only in the amount of broadly defined initial endowments, K_i, $i = 1, 2$. In particular,

$$K_1 = \lambda K, \qquad K_2 = (1 - \lambda)K, \tag{1}$$

where K is the total endowment in the economy and $\lambda \in (0, 1/2)$ captures the degree of initial inequality: group 1 is "poor" and group 2 is "rich." Assume for simplicity that

[11] For a similar perspective on envy, the rise of consumer culture, and the transition from envy-avoidance to "envy-provocation" see Belk (1995, chapter 1).

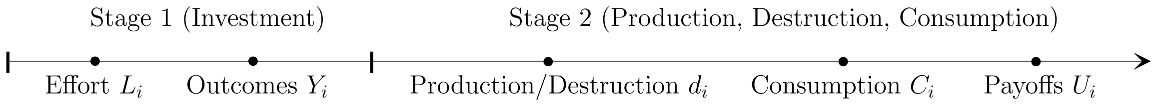


Figure 1: Timing of events in the envy game.

endowments remain constant over time and are fully transmitted across generations of the same dynasty, that is, $K_{it} = K_i$ for all t. This allows to abstract from changes in inequality due to bequest dynamics and concentrate on the role of opportunity-enhancing productivity growth.[12]

Each period, the two groups, or their representative agents, interact in the following two-stage game (see figure 1). In both stages each agent has a unit of time. In the first stage of the game this unit of time can be spent on investment (for instance, in education or innovation) and leisure. Specifically, each agent optimally allocates a certain fraction of his time, $L_{it} \in [0, 1]$, to produce an investment outcome, Y_{it}, according to

$$Y_{it} = F(K_{it}, L_{it}) = A_t K_{it} L_{it}, \quad i = 1, 2, \tag{2}$$

where A_t is the endogenous level of productivity, investment opportunities, or the stock of knowledge available to all individuals.[13] This outcome Y_{it} may be thought of as a factor of production like human or physical capital, intermediate product, or potential output in general, to be realized in the second stage. Time spent productively (effort) is costly in terms of leisure and causes disutility $e(L_{it}) = L_{it}$.

In the second stage, each agent allocates his unit of time between realization of own potential output (production) and disruption of the other agent's production process (destruction). Clearly, in this setup the only reason for spending time on destruction is envy. The model can be easily generalized to incorporate protection and theft, instead of pure destruction, without qualitatively affecting the main results.[14] If Agent i allocates a frac-

[12] An extension with bequest dynamics is presented in the supplementary online material.

[13] Linearity in A_t and K_{it} is inessential. Having exponents on these terms would just add more parameters to the model. Linearity in L_{it} allows to obtain closed-form solutions, but is not crucial for any of the qualitative results.

[14] The formulation with pure destruction allows to focus on envy as the *only* motive for disruptive behavior. In a setup with theft, envy is an additional force contributing to appropriation. The implications of protection in a model of appropriation (without envy) were examined by Grossman and Kim (1995; 1996). As will become clear, the setup with time allocation makes the model scale-free: optimal destruction intensity will depend on the inequality (but not the scale) of first-stage outcomes, which captures the essence of destructive envy.

tion $d_{it} \in [0,1]$ of his time to disrupt the productive activity of Agent j, the latter retains only a fraction p_{jt} of his final output, where

$$p_{jt} = p(d_{it}) = \frac{1}{1+\tau d_{it}}, \quad i,j = 1,2,\ i \neq j. \tag{3}$$

The function $p(d_{it})$ has standard properties: it is bounded, with $p(0) = 1$, decreasing, and convex (Grossman and Kim, 1995). Parameter $\tau > 0$ measures the effectiveness of destructive technology and may reflect the overall level of private property rights protection. In particular, property rights are secure if τ is low.

Time $1 - d_{it}$ is spent on the realization of potential output Y_{it} yielding the final output $(1 - d_{it})Y_{it}$. Since only a fraction $p(d_{jt})$ of this output is retained, the consumption level of Agent i is given by

$$C_{it} = (1 - d_{it})p(d_{jt})Y_{it}, \quad i,j = 1,2,\ i \neq j. \tag{4}$$

Finally, payoffs are generated. The utility function takes the following form:

$$U_{it} = U(C_{it}, C_{jt}, L_{it}) = v(C_{it} - \theta C_{jt}) - e(L_{it}) = \frac{(C_{it} - \theta C_{jt})^{1-\sigma}}{1-\sigma} - L_{it}, \tag{5}$$

where $i,j = 1,2$, $i \neq j$, $\sigma > 1$, and $\theta \in (0,1)$.[15] It is increasing in own consumption and decreasing in the other agent's consumption, as well as own effort. Agents are each other's reference points, which is natural in the setup with two individuals (groups), and parameter θ captures the strength of concern for relative standing (social comparisons). This utility function features additive comparison which is one of the two most popular ways to model envy, the other being ratio comparison.[16] Overall, the form of the utility function is identical to that in Ljungqvist and Uhlig (2000), except that in their model the reference point of each agent is the average consumption in the population. The latter assumption implies that each individual agent is too small to affect the reference consumption level, a common approach in models with interdependent preferences that

[15] Assume for simplicity that $U_{it} = -\infty$ whenever $C_{it} \leqslant \theta C_{jt}$. Under the assumption (10) below, this will never be the case in equilibrium. The assumption on the elasticity of marginal utility with respect to relative consumption, σ, is a convenient regularity condition that guarantees concavity of equilibrium outputs in endowments (see section 3.3). Linearity in effort is assumed for analytical tractability. For simplicity, we also abstract from leisure in stage 2.

[16] Additive comparison was assumed, among others, by Knell (1999) and Ljungqvist and Uhlig (2000). Boskin and Sheshinski (1978) and Carroll et al. (1997) are examples of models with ratio comparison. Clark and Oswald (1998) examine the properties of both formulations. The model in Gershman (2014) shows that the qualitative results of this section can be obtained in a framework with ratio comparison.

basically rules out the possibility of destructive envy. The present setup is thus different since groups (agents) are able to affect each other's outcomes.[17]

A crucial property of the utility function (5) is complementarity between own and reference consumption that leads to the "keeping up with the Joneses" kind of behavior, or emulation.[18] Clark and Oswald (1998) call such a function a "comparison-concave" utility (since v is concave). Intuitively, individuals are willing to match an increase in consumption of their reference group. The reason is that a rise in C_j reduces the relative consumption (status) of individual i which, under concave comparisons, increases the marginal utility of his own consumption. The payoff function (5) also implies that consumption is a "positional" good, while leisure (disutility of effort) is not. This view has been consistently advocated by Frank (1985; 2007) and finds support in the data (Solnick and Hemenway, 2005).

To complete the model, we need to define intergenerational linkages. In the basic version of the model generations are connected solely through the dynamics of investment opportunities, or productivity.[19] Specifically, productivity is driven by learning-by-doing and knowledge spillovers: the higher is the total investment outcome (potential output) in the society in a given period, the greater will be the stock of knowledge available for the next generation. That is, individual learning-by-doing affects future productivity of everyone in the economy. The simplest formulation capturing such dynamics, borrowed from Aghion et al. (1999), is

$$A_t = A(Y_{t-1}) = (Y_{1t-1} + Y_{2t-1})^{\alpha}, \tag{6}$$

where $\alpha \in (0, 1)$ is the degree of intergenerational knowledge spillover.[20] To account for external shocks to productivity, such as exogenous positive spillovers from a larger economy

[17]It is not uncommon in political economy literature to model group interaction in the context of two-agent games (Grossman and Kim, 1995). The implicit assumption is that groups are able to solve the collective action problem and act in a coordinated way.

[18]Dupor and Liu (2003) make a distinction between jealousy (envy) and KUJ behavior (emulation). The former is defined as $\partial U_{it}/\partial C_{jt} < 0$, while the latter is defined as $\partial^2 U_{it}/\partial C_{jt}\partial C_{it} > 0$. Interestingly, for a class of utility functions including (5) these two notions are equivalent.

[19]In the extension of the model available in the supplementary online material generations are linked through bequests.

[20]Aghion et al. (1999) assume that $\alpha = 1$. Here, $\alpha < 1$ in order to capture the possibility of a low productivity steady state, as will become clear from the analysis of section 4. If $\alpha = 1$ or A_t evolves according to a Romer-style "ideas production function," the society never gets stuck in a "bad" long-run equilibrium, but all the main qualitative results still carry through. We also assume for simplicity that the new knowledge is only available to the next generation, that is, there is no contemporary spillover effect.

described for the case of Tzintzuntzan in section 2.3, one could introduce an additional component in equation (6). For example, if $A_t = \zeta \cdot Y_{t-1}^{\alpha}$, a jump in $\zeta > 0$ might be interpreted as a shock to the process of knowledge creation.

3.2 Equilibria of the envy game

We start by analyzing the envy game for a given generation t. In what follows the time index t is omitted whenever this does not cause confusion. Given the dynamic structure of the game, we are looking for the subgame perfect equilibria. Hence, the model is solved backwards, starting at stage 2.

Second-stage equilibrium. Given the outcomes of the investment stage, Y_1 and Y_2, Agent i chooses the intensity of destruction, d_i, to maximize his payoff:

$$\max_{d_i} v((1-d_i)p(d_j)Y_i - \theta(1-d_j)p(d_i)Y_j) \qquad s.t. \quad 0 \leqslant d_i \leqslant 1, \tag{7}$$

where $i, j = 1, 2$, $i \neq j$. This yields the optimal second-stage response:[21]

$$d_i^* = \begin{cases} 0, & \text{if} \quad Y_i/Y_j \geqslant \tau\theta(1-d_j)(1+\tau d_j); \\ \frac{1}{\tau} \cdot \left(\sqrt{\tau\theta \frac{Y_j}{Y_i}(1-d_j)(1+\tau d_j)} - 1 \right), & \text{if} \quad Y_i/Y_j < \tau\theta(1-d_j)(1+\tau d_j). \end{cases} \tag{8}$$

Several features of this expression are of interest. First, without envy ($\theta = 0$) there is no destruction. If envy is present, the decision to engage in destruction depends on the inequality of investment outcomes, Y_i/Y_j. If Y_i is high enough relative to Y_j, Agent i finds it optimal to refrain from destruction. Otherwise, he allocates part of his time to destruction and its intensity is increasing in ex-post inequality, effectiveness of destructive technology, and the strength of envy. As such, this result reflects the trade-off between using the available unit of time productively and destructively to improve relative standing.

The product $\tau\theta$ is an endogenous (inverse) measure of tolerance for inequality. Given the ratio of investment outcomes, destructive envy is more likely to be activated if social comparisons are strong (large θ) and property rights are poorly protected (large τ). Hence, large $\tau\theta$ means low tolerance for inequality. If $d_j = 0$, the product $\tau\theta$ is the critical level of ex-post inequality, beyond which Agent i chooses to engage in destruction. Assume from now on that $\tau\theta < 1$, that is, if $d_j = 0$, there is some tolerance for inequality and $d_i^* = 0$ for $Y_i \geqslant Y_j$. Given this "non-zero tolerance for inequality" assumption, the agent with higher

[21] As shown in the proofs of the main results available online, the assumptions of the model guarantee that $C_i - \theta C_j > 0$ in equilibrium for $d_i^* < 1$. Hence, $d_i^* = 1$ is never optimal and this case is not considered.

investment outcome will never attack in the second-stage equilibrium, as established in the following lemma.[22]

Lemma 1. (Second-stage equilibrium). Let $Y_2 > Y_1$. Then, in the second-stage equilibrium $d_2^* = 0$ and

$$d_1^* = \begin{cases} 0, & \text{if} \quad Y_1/Y_2 \geqslant \tau\theta; \\ \frac{1}{\tau} \cdot \left(\sqrt{\tau\theta \frac{Y_2}{Y_1}} - 1 \right), & \text{if} \quad Y_1/Y_2 < \tau\theta. \end{cases} \tag{9}$$

Hence, Agent 2 retains a fraction $p_2^* = p(d_1^*) = \min\{\sqrt{Y_1/\tau\theta Y_2}, 1\}$ of his final output which is strictly decreasing in τ, θ, and Y_2/Y_1, if destruction takes place.

For expositional simplicity, we proceed as if there is a "predator and prey" type relationship (Grossman and Kim, 1996) between Agent 1 and Agent 2 from the outset, so that only the initially poor agent is allowed to engage in destruction at stage 2. Such assumption is made without loss of generality and underlines the asymmetric equilibrium roles played by the agents in the general formulation of the model. Specifically, proposition 1 will confirm that in the unique subgame perfect equilibrium of the game the initially better endowed agent always has a higher investment outcome and so, given lemma 1, would never spend time on destructive activities, even if permitted.

Note also that for Agent 1 to be able to avoid the negatively infinite utility, his choice of $d_1^* < 1$ has to yield positive relative consumption. This is secured by assuming that

$$\frac{K_1}{K_2} \equiv k > \underline{k} \equiv \frac{4\tau\theta}{(1+\tau)^2} \tag{10}$$

which imposes an upper bound on the level of initial inequality (or, equivalently, certain restrictions on parameters θ and τ) and makes the necessary "catching up" feasible, as formally demonstrated in the proof of lemma 3.

First-stage best responses. Agents are forward-looking and anticipate the optimal second-stage actions when making their first-stage decisions. Although Agent 2 is passive at stage 2, he is perfectly aware of how his investment outcome affects d_1^* and takes this into account at stage 1:

$$\max_{Y_2} U_2 = v(p(d_1^*)Y_2 - \theta(1-d_1^*)Y_1) - Y_2/AK_2 \qquad s.t. \quad 0 \leqslant Y_2 \leqslant AK_2. \tag{11}$$

[22]This assumption rules out the case in which the agent with higher investment outcome engages in destructive activities to improve his relative position even further. While such behavior is a theoretical possibility, the case considered here is more intuitive and consistent with the anecdotal evidence on destructive envy in section 2.

For technical reasons, it is easier to analyze the best responses (BRs) of both agents in terms of their consumption levels, C_i, rather than the first-stage outcomes Y_i. Note that these are different only if destruction actually takes place. In the latter case there is a one-to-one mapping between Y_i and C_i, as shown formally in the proof of lemma 2.[23]

Lemma 2. (BR of Agent 2). Denote the following productivity thresholds:

$$\widehat{A}_{21} \equiv \left(\frac{1}{1-\tau\theta^2}\right)^{\frac{\sigma}{\sigma-1}}, \quad \widehat{A}_{22} \equiv \widehat{A}_{21} \cdot \left(\frac{1+\theta^2}{2}\right)^{\frac{1}{\sigma-1}}, \quad \widehat{A}_{23} \equiv (1+\tau) \cdot \left(\frac{1+\theta^2}{2}\right)^{\frac{1}{\sigma-1}}.$$

Assume further that $\widehat{A}_{22} > \widehat{A}_{23}$ and $AK_2 > \widehat{A}_{23}$.[24] Then, the best-response of Agent 2, $\mathrm{BR}_2 \equiv C_2^*(C_1)$, is a piecewise function of the following form:

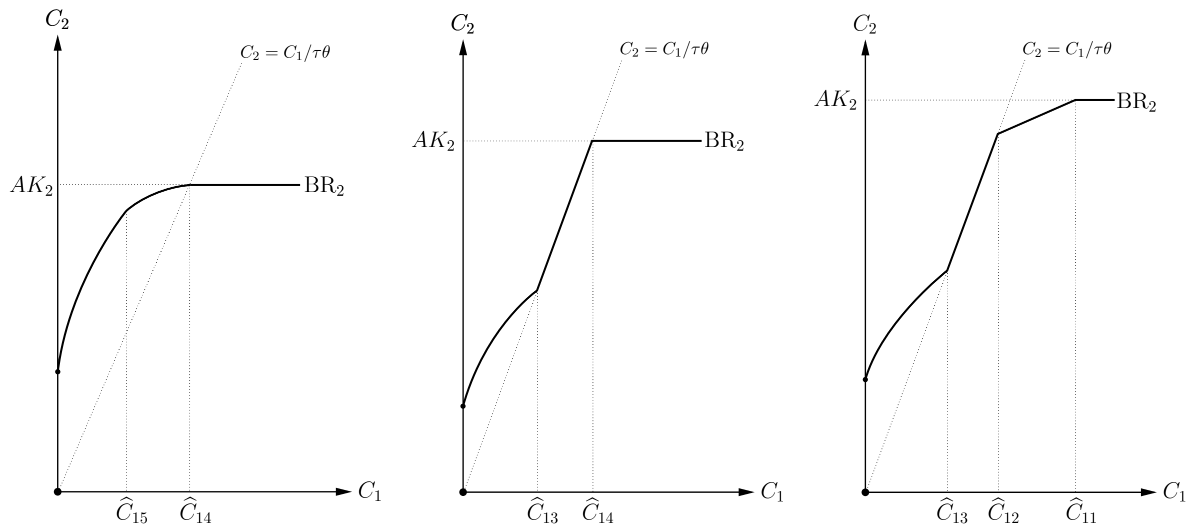
$$C_2^*(C_1) = \begin{cases} \min\{AK_2, \theta C_1 + (AK_2)^{1/\sigma}, C_1/\tau\theta\}, & \text{if} \quad C_1 \geqslant \widehat{C}_1; \\ \min\{C_2^d(C_1), \widetilde{C}_2^d(C_1)\}, & \text{if} \quad C_1 < \widehat{C}_1, \end{cases} \tag{12}$$

where $C_2^d(C_1)$ and $\widetilde{C}_2^d(C_1)$ are strictly increasing and concave. The definitions of $C_2^d(C_1)$, $\widetilde{C}_2^d(C_1)$, and the threshold level $\widehat{C}_1$, as well as the detailed form of (12), are given in the Appendix.

The threshold in expression (12) separates the cases in which Agent 2 allows second-stage destruction ($C_1 < \widehat{C}_1$) from those in which he does not ($C_1 \geqslant \widehat{C}_1$). In the former set of cases, the only difference between $C_2^d(C_1)$ and $\widetilde{C}_2^d(C_1)$ is whether Agent 2 invests "part time" ($L_2 < 1$) or "full time" ($L_2 = 1$) in stage 1. In the latter, more interesting set of cases, the best response of Agent 2 can be of three kinds: 1) invest full time ($C_2^* = AK_2$) hitting the resource constraint, 2) invest part time while displaying the KUJ behavior ($C_2^* = \theta C_1 + (AK_2)^{1/\sigma}$) and not being subject to destructive envy of the poor agent, 3) invest part time and produce the maximum output that rules out destructive envy ($C_2^* = C_1/\tau\theta$). Note that if destruction were not allowed (under perfect property rights protection), as in standard models of keeping up with the Joneses, and the resource constraint did not exist, the unique best response of Agent 2 would always be $C_2^* = \theta C_1 + (AK_2)^{1/\sigma}$. Thus, expression (12) highlights the additional constraints that Agent 2 faces: the resource (investment opportunities) constraint which must be satisfied and the "fear of envy constraint" which may or may not be binding.

[23]Technically, these consumption-based best responses correspond to the best responses in terms of first-stage investment outcomes adjusted for the second-stage destructive activity. Such transformation focuses on final outputs and makes it easier to analyze the destructive equilibria.

[24]The former assumption is made to cover all possible configurations of the best-response function. The latter imposes a lower bound on productivity which simplifies derivations.



(a) $\widehat{A}_{23} < AK_2 < \widehat{A}_{22}$ (b) $\widehat{A}_{22} \leqslant AK_2 < \widehat{A}_{21}$ (c) $AK_2 \geqslant \widehat{A}_{21}$

Figure 2: Best response of Agent 2.

Figure 2 depicts the alternative configurations of the second agent's best response.[25] The differences observed at various productivity levels deliver part of the intuition behind the upcoming main results. Specifically, the number of segments of the best-response function increases once AK_2 surpasses respective thresholds, opening qualitatively new possibilities.

For low levels of AK_2, as in figure 2(a), the optimal response is to invest full time, if $C_1 \geqslant \widehat{C}_{15}$, and consume either all of the output or the part that is left after destruction. For $C_1 < \widehat{C}_{15}$, it is optimal to invest part time in stage 1 while still being subject to destruction in stage 2. Note that for $C_1 < \widehat{C}_{14}$ it is never optimal to completely avoid destruction, even though it is always feasible. The intuition is that in this case full envy-avoidance would yield the level of relative consumption that is too low to be compensated by additional leisure time.[26]

Yet, as A increases, full envy-avoidance behavior becomes viable for $C_1 \in [\widehat{C}_{13}, \widehat{C}_{14})$, as depicted in figure 2(b). For such values of C_1 it is not worth allowing destruction at all: the gain in leisure is sufficient to compensate the reduction in relative consumption due to envy-avoidance. Clearly, if destructive activities on part of Agent 1 were not allowed,

[25]The values of thresholds $\widehat{C}_{11}$, $\widehat{C}_{12}$, $\widehat{C}_{13}$, $\widehat{C}_{14}$, and $\widehat{C}_{15}$ shown in figure 2 are specified in the detailed form of lemma 2 in the Appendix.

[26]In fact, for levels of C_1 between $\widehat{C}_{15}$ and $\widehat{C}_{14}$ Agent 2 would be willing to produce more than AK_2 to improve his relative position, but is constrained by the available resources.

Agent 2 would want to produce more than $C_1/\tau\theta$, but imminent destruction for C_2 above this threshold reduces the marginal benefit of investment and leads to self-restraint. Note also that for $C_1 > \widehat{C}_{14}$ Agent 2 wants to produce more than AK_2 but is constrained by the available time and investment opportunities.

Finally, for $AK_2 \geqslant \widehat{A}_{21}$ we observe all four possible actions of Agent 2. The new segment in figure 2(c) is the KUJ response for $C_1 \in [\widehat{C}_{12}, \widehat{C}_{11})$, in which case destructive envy is not binding and Agent 2 can produce as much as he really wants (absent the hazard of destruction) without crossing the $C_1/\tau\theta$ threshold. For $C_1 > \widehat{C}_{11}$ his envy-driven constructive behavior is constrained by the available resources.

Agent 1 is also forward-looking and knows his own optimal second-stage behavior when optimizing at the investment stage:

$$\max_{Y_1} U_1 = v((1 - d_1^*)Y_1 - \theta p(d_1^*)Y_2) - Y_1/AK_1 \qquad s.t. \quad 0 \leqslant Y_1 \leqslant AK_1. \tag{13}$$

Lemma 3. (BR of Agent 1). Denote the following productivity thresholds:

$$\widehat{A}_{11} \equiv \left(\frac{\tau}{\tau - 1}\right)^{\frac{\sigma}{\sigma-1}}, \quad \widehat{A}_{12} \equiv 1. \tag{14}$$

The best-response of Agent 1, $\text{BR}_1 \equiv C_1^*(C_2)$, is a piecewise function of the following form:

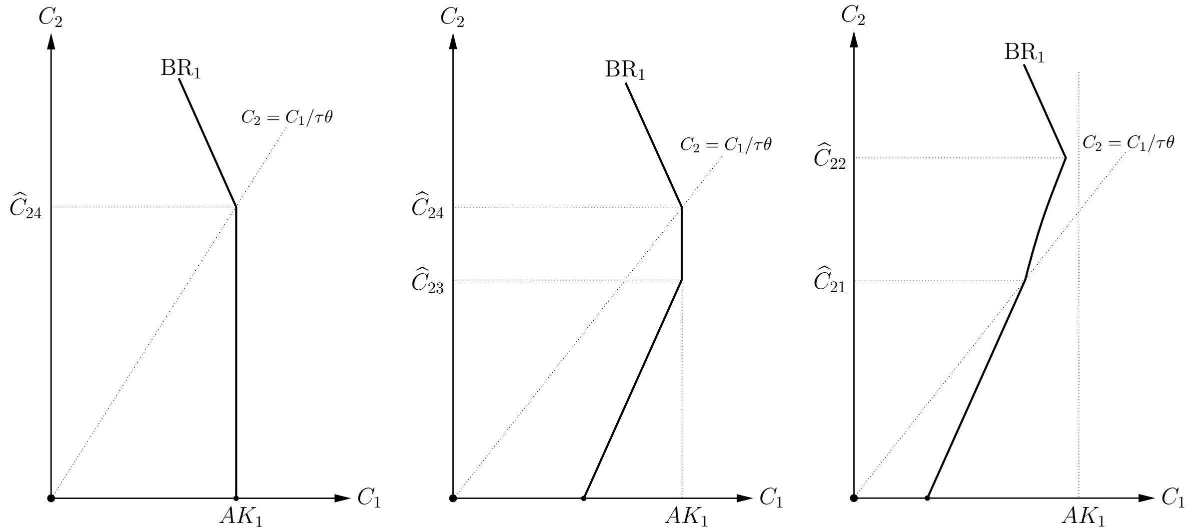
$$C_1^*(C_2) = \begin{cases} \min\{C_1^d(C_2), \widetilde{C}_1^d(C_2)\}, & \text{if} \quad C_2 \geqslant \widehat{C}_2; \\ \min\{AK_1, \theta C_2 + (AK_1)^{1/\sigma}\}, & \text{if} \quad C_2 < \widehat{C}_2, \end{cases} \tag{15}$$

where $C_1^d(C_2)$ is strictly increasing and convex and $\widetilde{C}_1^d(C_2)$ is linearly decreasing. The definitions of $C_1^d(C_2)$, $\widetilde{C}_1^d(C_2)$, and the threshold level $\widehat{C}_2$, as well as the detailed form of (15), are given in the Appendix.

The threshold in expression (15) separates the cases in which Agent 1 engages in envy-motivated destruction ($C_2 \geqslant \widehat{C}_2$) from those in which he does not ($C_2 < \widehat{C}_2$). As in the case of Agent 2, the only difference between $\widetilde{C}_1^d(C_2)$ and $C_1^d(C_2)$ is whether Agent 1 hits the resource constraint or not. If Agent 1 does not engage in destruction and chooses instead to catch up peacefully, his best response is $C_1 = \theta C_2 + (AK_1)^{1/\sigma}$ unless such KUJ-type behavior is constrained by the available investment opportunities AK_1.

As depicted in figure 3, the best response of Agent 1 becomes more "diverse" as productivity rises.[27] For low A, the best response is just to do as much as possible by investing full time. For $C_2 < \widehat{C}_{24}$, the output AK_1 is large enough to place ex-post inequality within

[27]The values of thresholds $\widehat{C}_{21}$, $\widehat{C}_{22}$, $\widehat{C}_{23}$, and $\widehat{C}_{24}$ shown in figure 3 are specified in the detailed version of lemma 3 in the Appendix.



(a) $AK_1 < \widehat{A}_{12}$ (b) $\widehat{A}_{12} \leqslant AK_1 < \widehat{A}_{11}$ (c) $AK_1 \geqslant \widehat{A}_{11}$

Figure 3: Best response of Agent 1.

the tolerance limit, but beyond that threshold it becomes optimal to engage in destruction in stage 2. For intermediate values of A we observe the emergence of the KUJ segment corresponding to $C_2 < \widehat{C}_{23}$: envy is satisfied through peaceful effort which is not constrained by the available resources. Yet, as C_2 increases, so does the desired level of C_1 until it hits the investment opportunities upper bound. Note that higher C_2 also increases the marginal benefit of second-stage destruction. For $AK_1 \geqslant \widehat{A}_{11}$ all three segments become apparent. The opportunities for peaceful catching up allow the KUJ segment to extend all the way to the $C_2 = C_1/\tau\theta$ border, beyond which Agent 1 produces below the tolerance threshold and engages in destruction in stage 2. Finally, at $C_2 = \widehat{C}_{22}$ the production level hits the capacity constraint and remains there for higher values of C_2.

Figure 4 shows the evolution of best responses driven by productivity growth from $A = A_0$ to $A = A_4$. The tendency for both functions is to become more "segmented" as investment opportunities increase and allow for a wider range of actions. For Agent 2, as figure 4(a) demonstrates, rising A leads to the emergence and subsequent expansion of the fear and KUJ segments. For Agent 1, as shown in figure 4(b), the main consequence of productivity growth is the emergence and extension of the KUJ segment. Thus, rising economic opportunities make the envy game more interesting: the latent regions of both best responses come into play potentially giving rise to new equilibrium outcomes. This dynamics is explored in detail in section 4.

Equilibria. To examine all possible equilibria of the envy game for a given generation we need to consider all cases in which the pairs of best responses are qualitatively different. Figure 5(a) shows a generic split of the (AK_1, AK_2) space according to these cases, including the condition $k < 1$ and the feasibility assumption $k > \underline{k}$.[28] Each ray from the origin corresponds to a particular level of inequality, while movements along such rays reflect productivity changes. The following proposition provides a taxonomy of all possible qualitatively different outcomes of the envy game for a given generation.

Proposition 1. (Equilibria of the envy game). There exists a unique subgame perfect equilibrium (C_1^*, C_2^*) of the envy game. It belongs to one of three classes:

1. Peaceful KUJ equilibria (KUJE):

$$C_i^* = \frac{(AK_i)^{1/\sigma} + \theta(AK_j)^{1/\sigma}}{1-\theta^2}, \quad i, j = 1, 2, \quad i \neq j; \tag{16}$$

$$C_1^* = AK_1, \quad C_2^* = \theta AK_1 + (AK_2)^{1/\sigma}; \tag{17}$$

$$C_i^* = AK_i, \quad i = 1, 2. \tag{18}$$

2. Peaceful fear-of-envy equilibria (FE):

$$C_1^* = \frac{\tau(AK_1)^{1/\sigma}}{\tau - 1}, \quad C_2^* = \frac{(AK_1)^{1/\sigma}}{\theta(\tau-1)}; \tag{19}$$

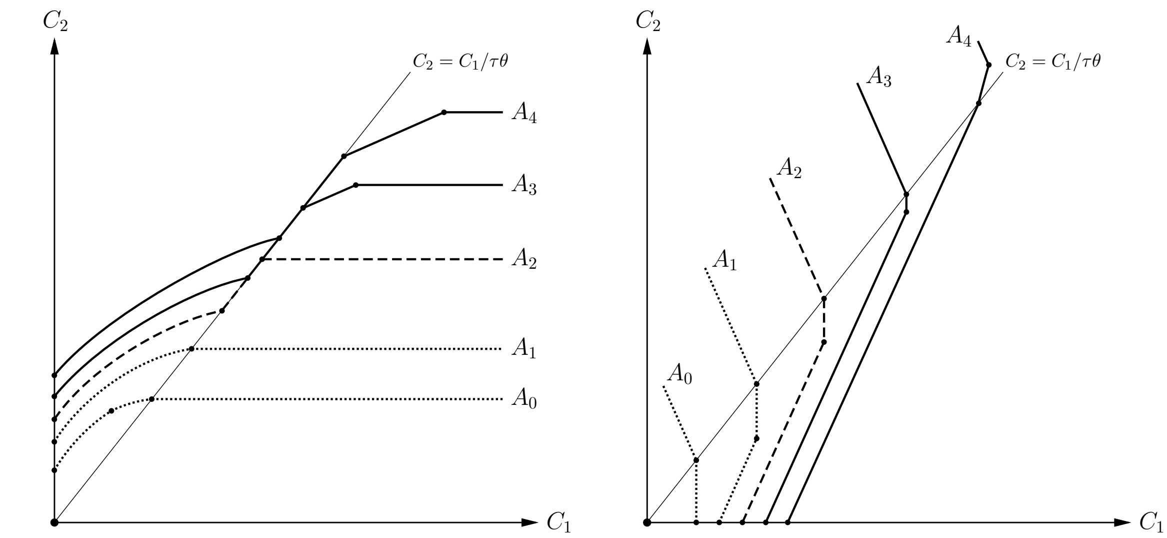
$$C_1^* = AK_1, \quad C_2^* = \frac{AK_1}{\tau\theta}. \tag{20}$$

3. Destructive equilibria (DE):

$$\begin{cases} C_1^* = C_1^d(C_2^*), \\ C_2^* = C_2^d(C_1^*); \end{cases} \quad \begin{cases} C_1^* = \widetilde{C}_1^d(C_2^*), \\ C_2^* = C_2^d(C_1^*). \end{cases} \quad \begin{cases} C_1^* = \widetilde{C}_1^d(C_2^*), \\ C_2^* = \widetilde{C}_2^d(C_1^*); \end{cases} \tag{21}$$

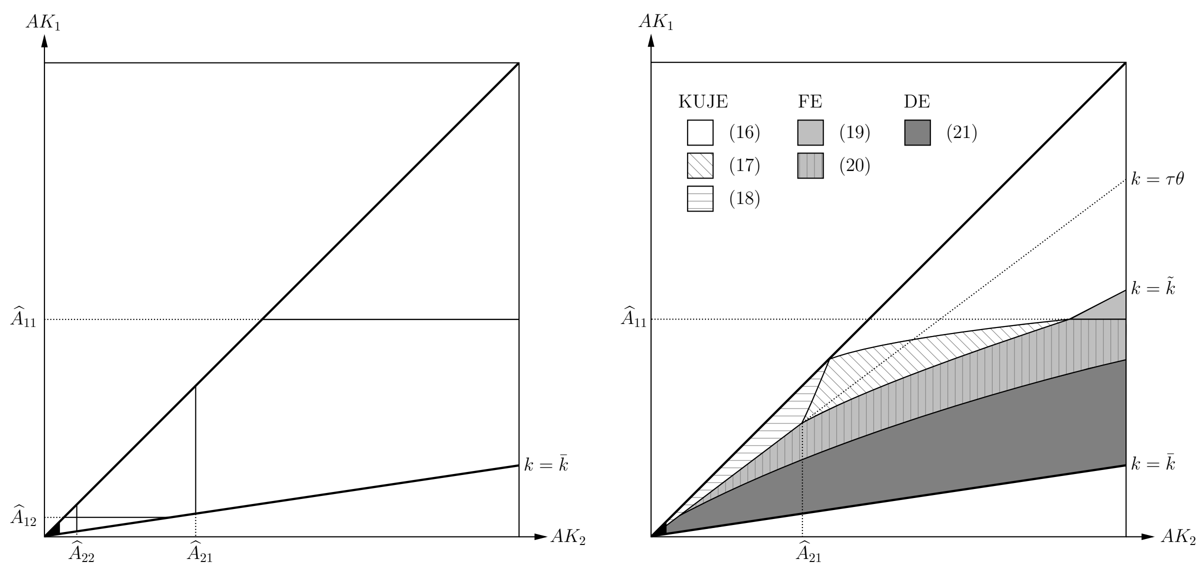
The exact conditions under which each equilibrium arises (the crucial inequality and productivity thresholds) are stated in the detailed form of the proposition in the Appendix.

[28] In particular, such split is typical under two assumptions: $\underline{k} < \hat{k}$ and $\widehat{A}_{22} > \widehat{A}_{23}$, where $\hat{k}$ is defined in the detailed form of proposition 1 in the Appendix. Both assumptions are maintained to achieve the highest possible variety of equilibria, with alternatives yielding only a subset of cases depicted in figure 5(a). The blackened southwestern corner of the sector is not considered due to an earlier technical assumption that $AK_2 > \widehat{A}_{23}$.



(a) Best responses of Agent 2 (b) Best responses of Agent 1

Figure 4: Productivity growth and the evolution of best responses.



(a) Regions with different pairs of BRs (b) Regions with different types of equilibria

Figure 5: The split of the (AK_1, AK_2) space according to pairs of BRs and equilibria.

Figure 5(b) graphically shows the regions corresponding to these qualitatively different classes of equilibria for all (AK_1, AK_2) pairs in the sector $\underline{k} < k < 1$, given the split of the space in figure 5(a). Regions with white background correspond to KUJ equilibria, the light gray area represents the fear-of-envy equilibria, and the dark gray color marks the destructive region. Figure 6 shows examples of equilibria for different levels of productivity.

In the first class of equilibria only constructive envy is present and both agents compete peacefully by undertaking productive investment in the first stage of the game, possibly hitting the resource constraint.[29] The features of a standard "keeping-up-with-the-Joneses" equilibrium (16) are well-known from previous literature and have been formally analyzed by Frank (1985) and Hopkins and Kornienko (2004), among others.[30] In particular, envy encourages additional investment effort which leads to "overworking" (see section 5).

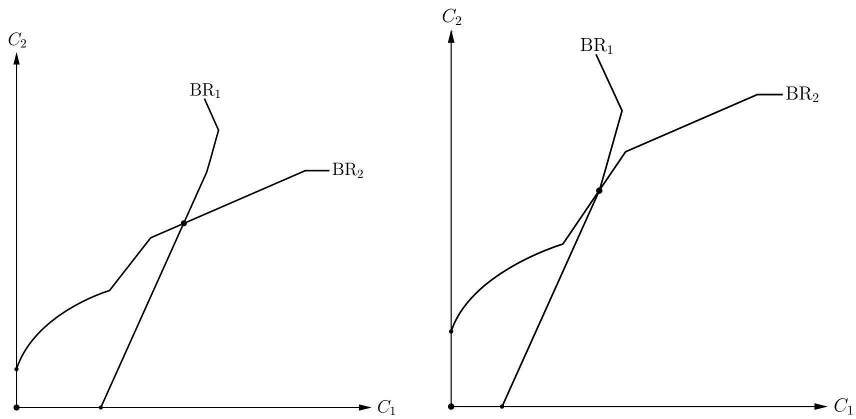
In the second class of equilibria the better endowed agent is constrained by the threat of envy-motivated destruction and invests the maximum possible amount of time that does not trigger aggression. There is no actual destruction in such equilibrium, but there is the "fear" of it that constrains the effort of Agent 2. This equilibrium resembles the fear of envy documented in many developing societies, as discussed in section 2.[31] In the third class of equilibria there is actual destruction and part of the time is used unproductively by Agent 1 to satisfy envy. As a consequence, part of the second agent's output is destroyed.

The intuition for when each kind of equilibrium emerges is simple. Given the level of productivity, a KUJ equilibrium is more likely vis-à-vis the fear or destructive outcomes if inequality is low and/or tolerance for inequality is high, that is, property rights are well-protected and social comparisons are weak. Otherwise, destructive envy is activated resulting in output loss due to underinvestment or outright destruction. The relation to inequality is obvious in figure 5(b): darker regions corresponding to cases in which destructive envy is either binding or present lie further away from the 45-degree line marking perfect equality. It is important to emphasize that, for a given level of productivity, three parameters (not counting σ) *jointly* determine the type of equilibrium. For instance, just

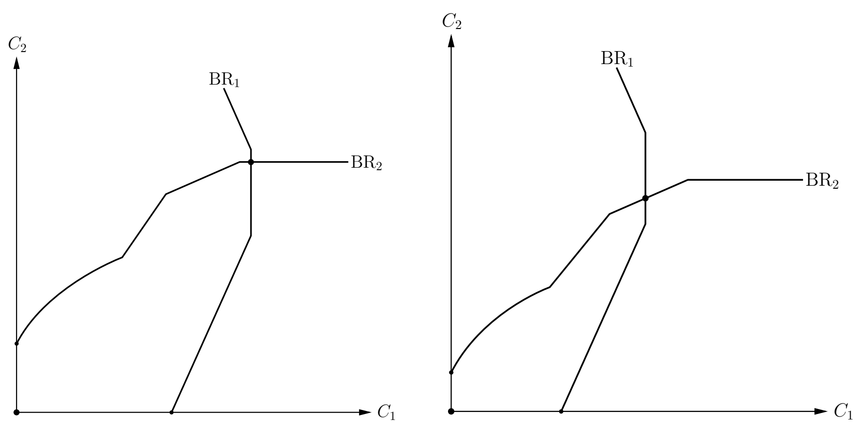
[29] We include the full-time investment case (18) in the group of KUJ-type equilibria, since it does not feature either destructive envy or the fear of it. One caveat, however, is that at very low levels of productivity working full time is unrelated to KUJ-type incentives. With this in mind, by default we refer to case (18) as the one in which it is the catching-up behavior that is limited by the available resources.

[30] Note that (16) would always be the unique equilibrium of the envy game in the absence of destructive technology (under perfect property rights protection) and the resource constraint.

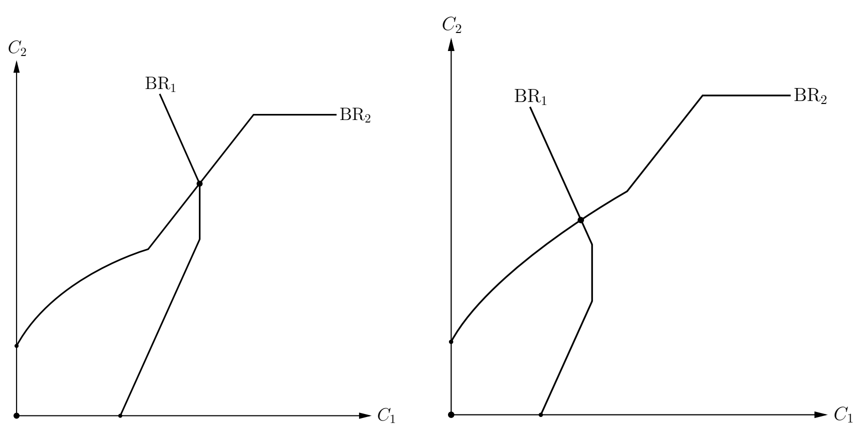
[31] Mui (1995) constructs a theoretical framework in which (costless) technological innovation may not be adopted in anticipation of envious retaliation. The intuition of the fear equilibrium in the present theory is similar, except that here the fear of envy operates on the intensive margin by discouraging (costly) investment. Furthermore, in Mui's framework retaliation reduces envy directly by assumption rather than through the improvement of the relative standing of the envier. Finally, his paper ignores constructive envy and thus, focuses on one side of the big picture.



(a) $AK_1 \geqslant \widehat{A}_{11}$, $AK_2 \geqslant \widehat{A}_{21}$: KUJE (left) and FE (right)



(b) $AK_1 < \widehat{A}_{11}$, $AK_2 \geqslant \widehat{A}_{21}$: KUJE



(c) $AK_1 < \widehat{A}_{11}$, $AK_2 < \widehat{A}_{21}$: FE (left) and DE (right)

Figure 6: Equilibria of the game for different productivity levels.

having low inequality is not enough to be in the KUJE. If at the same time institutions are very weak (destructive technology is efficient) and/or relative standing concerns are very strong, the society may still end up in a fear or even destructive equilibrium.

A question of special interest is how endogenously rising productivity can affect the type of equilibrium, given the level of inequality and tolerance parameters. First, as follows from the detailed form of proposition 1 in the Appendix, once A gets large enough, the type of equilibrium is determined only by k, τ, and θ. Thus, rising productivity need not automatically turn destructive envy into constructive if there is persistently high inequality or low tolerance for inequality. For example, for $k < \tilde{k}$ any level of productivity yields either fear-type or destructive equilibrium.[32] Conversely, if $k > \tau\theta$, the equilibrium outcome is always a KUJ-type competition, and growing A just takes the society from a full-time equilibrium, in which individuals are constrained by the available resources, to a conventional KUJE with positively sloped best responses, as in the left panel of figure 6(a).

The most interesting case is the one with intermediate levels of inequality (or tolerance for inequality), that is $\tilde{k} < k < \tau\theta$. For such parameter values, productivity growth can lead the society through all envy regimes, from destructive equilibrium through the fear of envy and to the KUJ competition, consistent with the evidence on such transition presented in section 2.3.[33] As will become clear from the analysis of section 4, even when productivity matters for determination of the equilibrium type, inequality and tolerance parameters play a key role in guiding the long-run dynamics of the society. Before looking at the evolving role of envy in the process of development, it is instructive to first consider comparative statics for each particular equilibrium type.

3.3 Comparative statics

Consider the impact of four parameters of interest, λ, θ, τ, and A, on economic performance as measured by final outputs. The qualitative effects are similar within the three groups of equilibria identified above.

As follows from proposition 1, total outputs in the KUJ-type equilibria (16), (17), and (18) are given by $Y = (\lambda^{1/\sigma} + (1-\lambda)^{1/\sigma}) \cdot (AK)^{1/\sigma}/(1-\theta)$, $Y = (1+\theta)\lambda AK +$

[32] This is due to the assumption that $\underline{k} < \hat{k}$. If, on the other hand, $\underline{k} > \tilde{k}$, a high enough level of productivity guarantees a peaceful KUJ equilibrium. The thresholds $\tilde{k}$ and $\hat{k}$ are given by (28).

[33] A shift towards this sector from higher starting levels of inequality may happen endogenously due to bequest dynamics, see the online supplementary material.

$(1-\lambda)^{1/\sigma}(AK)^{1/\sigma}$, and $Y = AK$, respectively. Clearly, economic performance in these cases does not depend on τ since destructive envy is not binding. The effect of θ is straightforward: increasing the strength of relative concerns acts as additional incentive to work, which leads to higher levels of effort and output in the conventional KUJ equilibrium (16). In the "partial" KUJ equilibrium (17) we observe a similar effect only for Agent 2, while Agent 1 is working at the capacity constraint and cannot respond to rising θ. Finally, in case (18) both agents invest at their capacity levels and the incentivizing marginal effect of envy is absent.

The effect of raising λ (increasing equality) on economic activity depends crucially on σ, with the exception of the full-time equilibrium (18), in which redistribution alters individual investment levels, but not aggregate output. Under the baseline assumption $\sigma > 1$, outputs in the conventional KUJ equilibrium are strictly concave and increasing functions of λ which is more natural than a kind of nondecreasing returns to scale that would emerge under $\sigma \leqslant 1$. The total effect of redistribution on private outputs in (16) consists of two parts: wealth effects and comparison effects. Wealth effects are just the direct effects of making one agent poorer and the other richer. In case of increasing λ the wealth effect is positive for Agent 1 and negative for Agent 2. The total wealth effect on output is positive since the poor agent is more productive on the margin under concave output functions.[34] Comparison effects reflect the fact that the reference group becomes poorer for Agent 1 and richer for Agent 2. Consequently, comparison effect is negative for Agent 1 and positive for Agent 2. The total comparison effect has the same sign as the total wealth effect. In particular, under $\sigma > 1$, the negative comparison effect on the output of the poor is outweighed by the positive comparison effect on the output of the rich. In the partial KUJ equilibrium (17) Agent 2 experiences both wealth and comparison effects, while Agent 1 is investing at full capacity. As shown in the proof of proposition 2, the effect of λ on aggregate output is always positive in this case, that is, the negative wealth effect on the rich is always dominated by the incentivizing effect of redistribution.

Next, consider the fear-type equilibria (19) and (20) in which total output is given, respectively, by $Y = (1+\tau\theta)(\lambda AK)^{1/\sigma}/(\theta(\tau-1))$ and $Y = (1+\tau\theta)\lambda AK/(\tau\theta)$. In both cases the output of Agent 2 is "tied" to that of Agent 1 with the coefficient of tolerance $1/\tau\theta$. Raising τ (reducing the quality of institutions) decreases the tolerance of Agent 1 for inequality and aggravates the "fear constraint" of Agent 2. This means that with

[34] This "opportunity-enhancing" effect of redistribution under diminishing returns to individual endowments and imperfect capital markets is well-known in the literature, see, for example, Aghion et al. (1999, section 2.2).

higher τ Agent 2 has to produce less to avoid destructive envy, which leads to lower individual and total outputs. The effect of raising θ is similar since it, too, decreases the tolerance for inequality. This is in stark contrast with the role of envy in the KUJ-type equilibria. In the latter case envy acts as additional incentive to work, while in the fear equilibria it constrains productive effort by increasing the hazard of destructive envy. On the other hand, the effect of raising equality in the fear equilibria is unambiguously positive. Increasing λ enhances the output of Agent 1 which "trickles up" into the higher output of Agent 2. That is, redistribution from the rich to the poor increases investment and final output by alleviating the fear of envy constraint.

Destructive equilibria are harder to examine analytically. Multiple effects are at work which makes the aggregate comparative statics with respect to θ ambiguous for the first case in (21). If inequality is high or tolerance for inequality is low, stronger envy leads to substantial destruction which may lower the consumption of Agent 2, as well as the total final output, C. In contrast, if the destructive environment is not severe (or σ, the catching-up propensity, is high), the stimulating effect of envy dominates. Thus, comparative statics in the DE combines the features of the FE and the KUJE. At the same time, higher τ and lower λ unambiguously decrease total consumption.

Finally, rising productivity of investment, A, has a positive impact on aggregate economic performance, regardless of the equilibrium type. The following proposition summarizes the comparative statics results discussed above.

Proposition 2. (Comparative statics of the envy game). The effects of parameters on the aggregate economic performance are as indicated in table 1.

Table 1: The effects of parameters on aggregate final output

	KUJE			FE		DE		
	(16)	(17)	(18)	(19)	(20)	(21):1	(21):2	(21):3
λ	+	+	0	+	+	+	+	+
θ	+	+	0	−	−	±	−	−
τ	0	0	0	−	−	−	−	−
A	+	+	+	+	+	+	+	+

Overall, setting aside the "uninteresting" equilibrium (18) and the ambiguity with respect to θ for the first case in (21), the key messages of proposition 2 are the following:

1) equality and higher productivity increase aggregate final output; 2) the strength of envy enhances output when society is in the unconstrained KUJ-type competition and has a detrimental effect whenever destructive envy and the fear of it are active; 3) better property rights enhance economic activity when they matter, that is, when destructive envy is binding.

4 Envy and the growth process

Now that we established how incentives respond to changes in parameters in each equilibrium, we turn to the analysis of dynamics implied by proposition 1 together with equation (6), that is, transitions from one equilibrium type to another driven by endogenous productivity growth. Specifically, focus on the sector defined by $k \in (\tilde{k}, \tau\theta)$, in which such growth can trigger qualitative changes. Clearly, at each point in time the dynamics is determined by the region of the phase plane 5(b) in which the society is located.

Assume that the initial level of productivity A_0 is large enough, so that the society is already past the destructive region and starts in the fear-type equilibrium (20).[35] Then, the dynamics of productivity is given by

$$A_t = \begin{cases} [(1+1/\tau\theta)K_1]^\alpha \cdot A_{t-1}^\alpha, & \text{if} \quad A_{t-1} < \hat{A}; \\ [(1+\theta)A_{t-1}K_1 + (A_{t-1}K_2)^{1/\sigma}]^\alpha, & \text{if} \quad \hat{A} \leqslant A_{t-1} < \tilde{A}; \\ [(K_1^{1/\sigma} + K_2^{1/\sigma})/(1-\theta)]^\alpha \cdot A_{t-1}^{\alpha/\sigma}, & \text{if} \quad A_{t-1} \geqslant \tilde{A}, \end{cases} \tag{22}$$

where the productivity thresholds are

$$\hat{A} \equiv \left[\frac{\tau\theta}{1-\tau\theta^2} \cdot \frac{K_2^{1/\sigma}}{K_1}\right]^{\frac{\sigma}{\sigma-1}}, \quad \tilde{A} \equiv \left[\frac{K_1^{1/\sigma} + \theta K_2^{1/\sigma}}{(1-\theta^2)K_1}\right]^{\frac{\sigma}{\sigma-1}},$$

as follows directly from proposition 1. In the long run, the society will settle in one of the three regions, as established in the following proposition.

[35] Another reason for why the economy might start in the fear-type equilibrium is if it has redistributive mechanisms in place. A variation of the basic model in Gershman (2012) shows how in the presence of preemptive transfers destructive equilibrium can be replaced by a "fear equilibrium with transfers."

Proposition 3. (Long-run steady state.) Assume that $k \in (\tilde{k}, \tau\theta)$ and $A_0 \in (\bar{A}, \hat{A})$.[36] Then

$$\lim_{t\to\infty} A_t = \begin{cases} A_1^* \equiv [(1+1/\tau\theta)K_1]^{\frac{\alpha}{1-\alpha}}, & \text{if} \quad \alpha < \hat{\alpha}; \\ A_2^*, & \text{if} \quad \hat{\alpha} \leqslant \alpha < \tilde{\alpha}; \\ A_3^* \equiv [(K_1^{1/\sigma} + K_2^{1/\sigma})/(1-\theta)]^{\frac{\alpha\sigma}{\sigma-\alpha}}, & \text{if} \quad \alpha \geqslant \tilde{\alpha}, \end{cases} \tag{23}$$

where A_2^* is implicitly defined by equation $(A_2^*)^{1/\alpha} = (1+\theta)A_2^* K_1 + (A_2^* K_2)^{1/\sigma}$, the thresholds $\hat{\alpha}$ and $\tilde{\alpha}$ are implicitly defined by equations $A_1^* = \hat{A}$ and $A_2^* = \tilde{A}$, respectively. Furthermore, $\hat{\alpha}_\theta > 0$, $\hat{\alpha}_\lambda < 0$, $\hat{\alpha}_\tau > 0$, and $\tilde{\alpha}_\theta > 0$, $\tilde{\alpha}_\lambda < 0$, $\tilde{\alpha}_\tau = 0$.

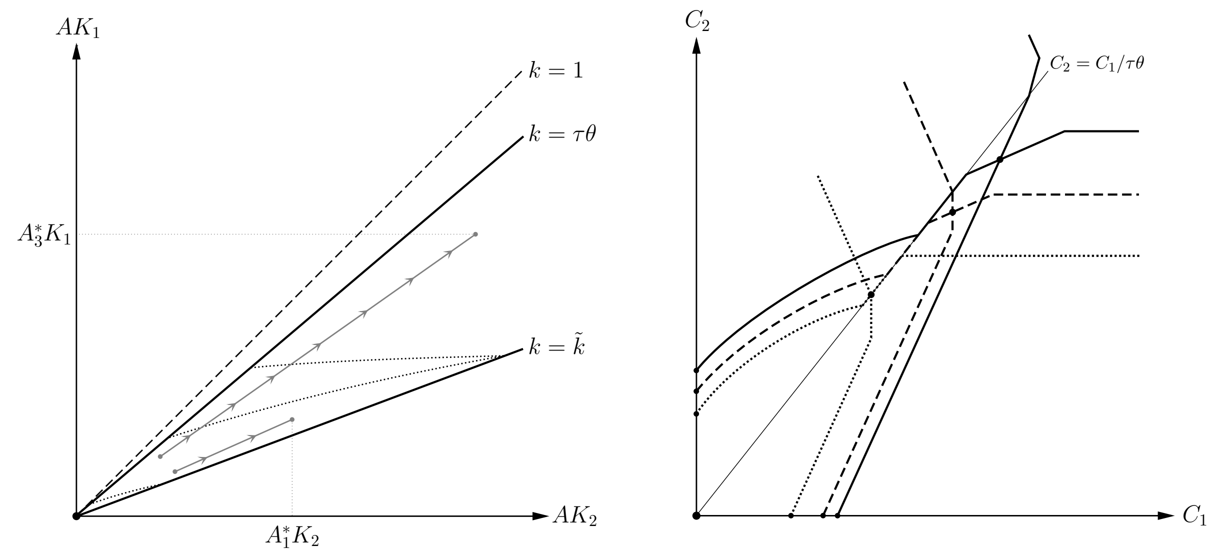
Proposition 3 implies that lower inequality and higher tolerance for inequality are likely to put the society on a trajectory that leads to the long-run KUJ equilibrium marked by high productivity level. In contrast, under high fundamental inequality and low tolerance the society is likely to get stuck in the fear of envy region with low productivity in the steady state. The comparative statics of long-run levels of productivity clearly replicate those in proposition 2.

Figure 7 demonstrates how endogenous productivity growth can take the society away from the initial fear equilibrium to the long-run KUJE. The left panel shows such transition on the (AK_1, AK_2) plane, while the right panel shows the evolution of best responses and the equilibrium type following productivity growth. Initially, peaceful investment opportunities are scarce locating the society in the fear equilibrium. Yet, the growth process expands investment opportunities, and envy-avoidance behavior, dictated by the destructive side of envy, paves the way to constructive emulation. Note also that ex-post inequality of outputs reduces as a result of this transition.[37] If, however, the society starts off closer to the $k = \tilde{k}$ line, it is likely to stay in the same fear region where it started, as the lower trajectory in figure 7(a) illustrates. In such cases, the fear of envy does not let investment opportunities grow big enough to enable full-fledged constructive KUJ competition. Apart from inequality and tolerance parameters, the development trajectory of the society might be affected by an external shock to investment opportunities of the type discussed in section 2.3 or a change in the strength of knowledge spillover α.

Importantly, depending on the current regime, envy affects economic growth in opposite ways. In the fear-type equilibrium it discourages investment and growth, while in the KUJ-type equilibria the effect is the opposite. The same is true for the long-run levels of

[36] The lower bound on initial productivity is $\bar{A} \equiv [\tau\theta/((1-\tau\theta^2)K_1)]^{\sigma/(\sigma-1)} \cdot [(1+\theta^2)K_2/2]^{1/(\sigma-1)}$.

[37] Specifically, as follows directly from proposition 1, ex-post inequality is equal to $C_1/C_2 = \tau\theta$ in the fear region, then monotonically decreases to $(k^{1/\sigma} + \theta)/(1 + \theta k^{1/\sigma})$ and stays at that level thereafter.



(a) Alternative growth trajectories

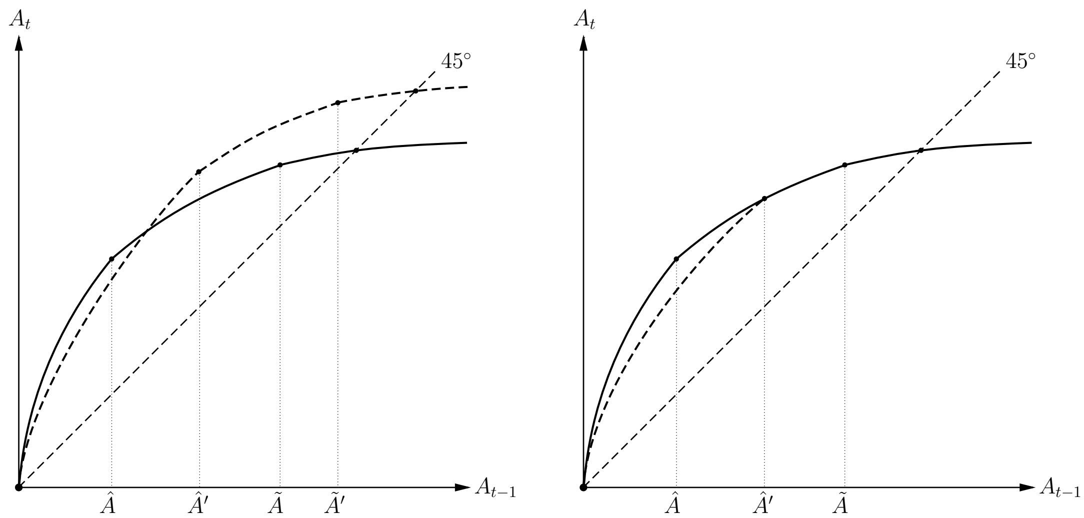
(b) Dynamics of best responses

Figure 7: Productivity-driven transition from FE to KUJE.

productivity. A number of factors, such as religion and ideology in general, can plausibly affect the intensity of social comparisons.[38] In the context of the model, religious and moral teachings condemning envy cause downward pressure on θ. In the fear region, such teachings enhance growth because the fear constraint of the rich is alleviated permitting higher effort. Moreover, a fall in θ lowers the thresholds $\hat{A}$ and $\tilde{A}$ contributing to a faster transition from FE to KUJE. As the economy enters the KUJ region, destructive envy turns into emulation, and θ has the opposite impact on economic performance. In the KUJ region the same factors that drive the society out of the fear equilibrium have a negative effect on output.

An example of ideology positively affecting θ is that of material egalitarianism. The concept of everyone being equal and the neglect of private property rights are effective in fostering social comparisons and lowering tolerance for inequality. Hence, this ideology operates in favor of destructive envy in the fear region and delays the transition to the KUJE,

[38]All major world religions denounce envy. In Judeo-Christian tradition envy is one of the deadly sins and features prominently in the tenth commandment. Schoeck (1969, p. 160) goes as far as to say that "a society from which all cause of envy had disappeared would not need the moral message of Christianity."



(a) Dynamic effects of a rise in θ (b) Dynamic effects of a rise in τ

Figure 8: Envy, property rights, and productivity dynamics.

as shown in figure 8(a).[39] Yet, upon entering the KUJ region, intense social comparisons are channeled into productive activity and boost economic growth.

Similarly, one could examine the dynamic effects of a shock in τ that could be due to various reasons, from changes in technologies of destruction and protection to reforms of legal institutions. Better property rights encourage productivity growth in the fear region and have no effect on either the dynamics or the steady state in the KUJ region. In contrast, an increase in τ would endogenously prolong the presence of the economy in the fear region, as shown in figure 8(b), while at the same time making the FE more egalitarian, since the erosion of institutions decreases tolerance for inequality and exacerbates the fear constraint. This may explain the persistence of the fear of envy, along with such characteristics as poorly protected private property rights and relatively low inequality.

5 Welfare, property rights, and inequality

As shown in the previous section, the initial level of inequality and the two tolerance parameters jointly determine the growth trajectory of the economy, potentially resulting in qualitatively different long-run equilibria. Yet, in principle, societies might have the

[39]Figure 8 ignores for simplicity that for low enough A the dynamics is governed by the outcomes of destructive equilibria.

ability to challenge the underlying institutional and distributional status quo if this leads to welfare improvements. This section examines the incentives to undertake such changes.

Specifically, assume that $k \in [\hat{k}, \tilde{k})$ and the society is in the long-run fear equilibrium with the steady-state productivity level A^{F} given by

$$A^{\mathrm{F}} = \left[\frac{1+\tau\theta}{\theta(\tau-1)} K_1^{1/\sigma} \right]^{\frac{\alpha\sigma}{\sigma-\alpha}}, \tag{24}$$

where, in addition, $A^{\mathrm{F}} K_1 \geqslant \hat{A}_{11}$ and $A^{\mathrm{F}} K_2 \geqslant \hat{A}_{21}$.[40] Given this starting point, when will both social groups want to move away from the fear of envy equilibrium to a KUJ trajectory by adopting better institutions (lower τ) or redistributing the initial wealth from the rich to the poor (higher k)? In answering this question we consider both the incentives of the current generation (short run) and the welfare consequences for future generations of both dynasties upon convergence to the new long-run KUJE.[41] In that steady state the level of productivity is given by

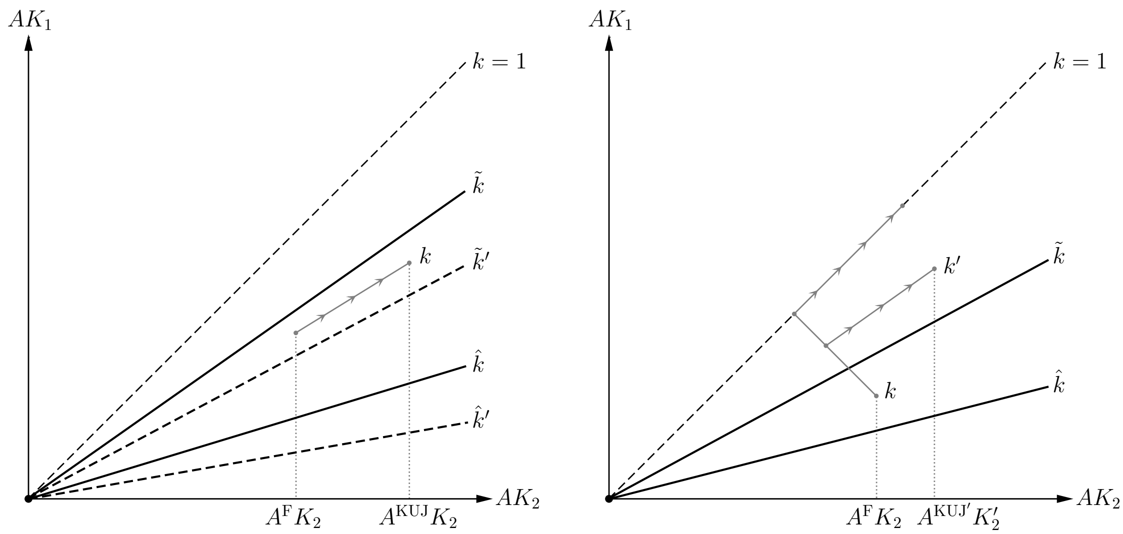
$$A^{\mathrm{KUJ}} = \left[\frac{K_1^{1/\sigma} + K_2^{1/\sigma}}{1-\theta} \right]^{\frac{\alpha\sigma}{\sigma-\alpha}}, \tag{25}$$

which is always greater than A^{F}. Note that only future generations will be able to reap the full benefit of technological advancement prompted by institutional or distributional change.

The two thought experiments are illustrated in figure 9. Figure 9(a) shows a switch to KUJ dynamics via the adoption of better institutions: a lower value of τ changes the split of the phase plane, as a result of which the society finds itself in the KUJ region and starts growing towards the new long-run steady state. Figure 9(b) shows the consequences of an ex-ante redistribution of endowments: a jump from k to k' does not affect the split of the phase plane into sectors but puts the society onto a more equal growth path in the KUJ region leading to a higher long-run productivity level. We examine the details of both scenarios in turn.

[40] That is, for concreteness we focus on case 19, although similar intuition would clearly hold for (20). The expression for A^{F} follows from equations (6) and (19).

[41] An alternative, but similar way to think about it would be to consider a benevolent utilitarian social planner maximizing the (discounted) welfare of current and all future generations of the society. A more involved option would be to incorporate Barro-style dynastic preferences in the analysis. In any case the crucial point will be the differential welfare effect on current versus future generations.



(a) Institutional change: a decrease in τ

(b) Redistribution: an increase in k

Figure 9: From fear of envy to keeping up with the Joneses.

5.1 Institutional change

The connection between institutions and externalities was famously drawn by Demsetz (1967) who considered the emergence of property rights to be a way to cope with certain externality problems. In the present theory, two types of externalities make institutions matter: a negative consumption externality (envy) and a positive intergenerational learning-by-doing externality (knowledge spillover). As established in the following proposition, on the one hand, *weaker* property rights protection may internalize the negative consumption externality. On the other hand, *stronger* property rights allow future generations to benefit from the positive knowledge spillover which has a positive effect on social welfare in the long run. Thus, it is the relative strength of the two types of externalities that determines the long-run welfare effect of an institutional change.

Proposition 4. (Welfare and institutional change.) Assume that $k \in [\hat{k}, \tilde{k})$ and the society is in the long-run FE. Then,

I. *Short run* (current generation). $\exists!\ \bar{\theta} \in (0, 1)$ such that: 1) If $\theta > \bar{\theta}$, a shift to the KUJE via better institutions is a Pareto worsening; 2) If $\theta \leqslant \bar{\theta}$, $\exists!\ \bar{k}_1 \in [\hat{k}, \tilde{k})$ such that this shift is a Pareto worsening if and only if $k > \bar{k}_1$.

II. *Long run* (future generations). $\exists!\ \bar{\alpha} \in (0, 1)$ such that: 1) If $\alpha < \bar{\alpha}$, the long-run KUJE is never a Pareto improvement over the initial fear equilibrium; 2) If $\alpha \geqslant \bar{\alpha}$,

$\exists! \; \bar{k}_2 \in [\hat{k}, \tilde{k}]$, such that the long-run KUJE is a Pareto improvement over the initial fear equilibrium if and only if $k > \bar{k}_2$.

Consider the short-run part first. The intuition for this result revolves around the negative effect of the consumption externality on social welfare. As follows from the proof of proposition 4, Agent 1 (the poor) always prefers the FE: he enjoys the same level of relative consumption as in the KUJE, but exerts less effort. Agent 2 (the rich) always benefits from higher relative consumption in the KUJE, but at a cost of increased effort, and thus, faces a trade-off. It turns out that he prefers to stay in the FE if only if in the alternative KUJE he would have to work too hard to support his high relative standing, or, in other words, if a rise in social status does not compensate for the cost of foregone leisure. That would be the case if either social comparisons are strong (θ is large) or inequality is low (k is large). Under these conditions the FE is Pareto dominant since the fear of destructive envy restrains effort and curbs the consumption externality that otherwise leads to overworking in the KUJE. Thus, weaker property rights protection corrects the distortion caused by envy.[42]

As follows from proposition 2, regardless of welfare effects, outputs of both agents will always be higher in the KUJE compared to the initial FE. The result that both agents can be better off in an equilibrium with lower consumption is reminiscent of what Graham (2010) calls the "paradox of happy peasants and miserable millionaires" and related research in happiness economics (Clark et al., 2008).

It is also instructive to read the short-run part of proposition 4 in the "reverse" order, assuming that the society is initially in the KUJE. Then, it implies that, under strong enough consumption externality, weaker property rights protection (higher τ) causing a switch to the fear equilibrium might constitute a Pareto improvement. For low values of θ, such shift would be favored by the rich if and only if the distribution of endowments is relatively equal. Such reading of this result can be viewed as a formalization of a more than a century-old argument raised by Veblen (1891) in an attempt to explain the support for socialist movement and the abolition of private property rights. In particular, Veblen argued that the "ground of the unrest with which we are concerned is, very largely, jealousy, – envy, if you choose: and the ground of this particular form of jealousy that makes for socialism, is to be found in the institution of private property."

[42]Curiously, this is akin to the effects of Pareto improving redistributive taxation in the presence of concern for relative standing, see Boskin and Sheshinski (1978), Oswald (1983), and Frank (1985).

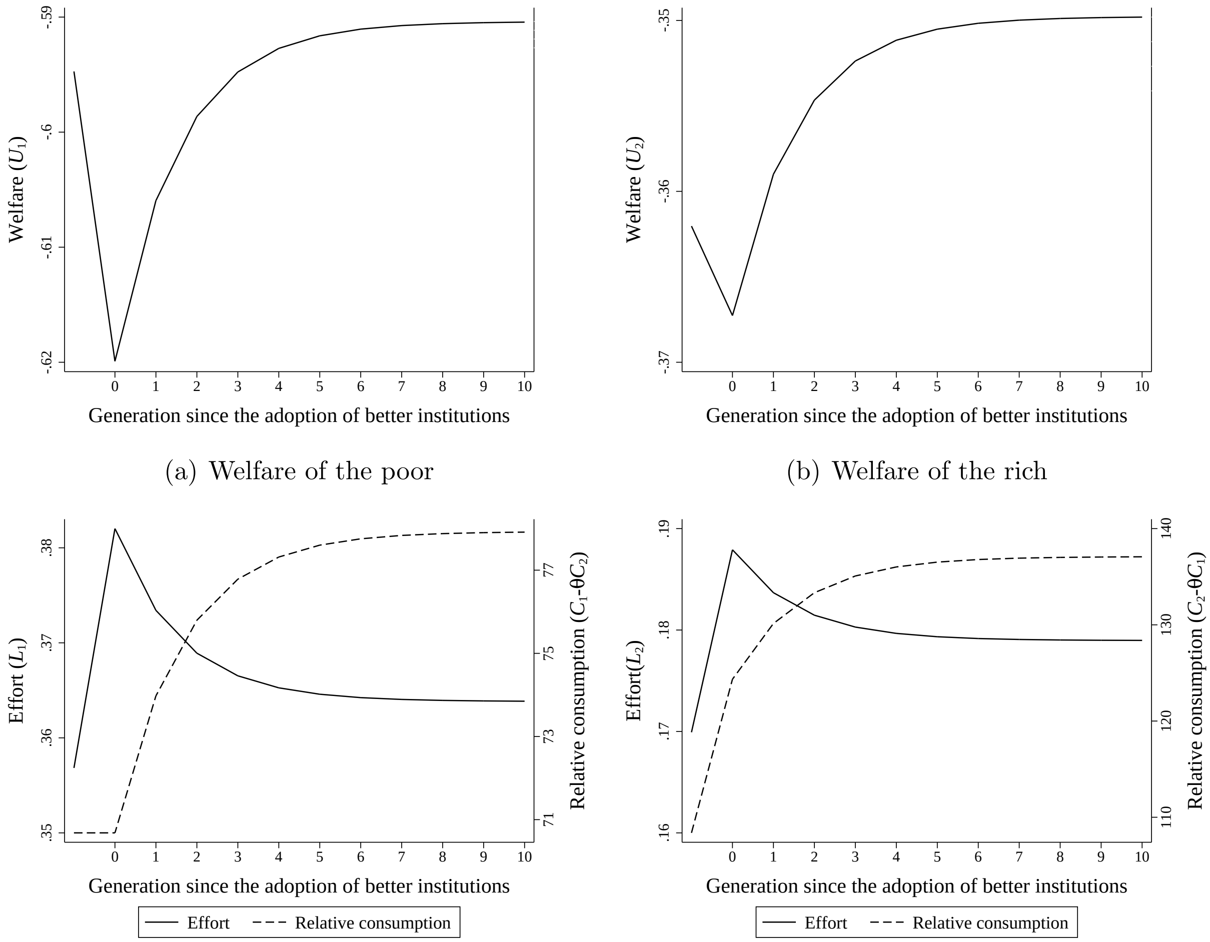
Veblen goes on to describe what in the language of the present theory represents a transition from the KUJE ("keep up appearances") to the FE ("socialism"): "The ultimate ground of this struggle to keep up appearances by otherwise unnecessary expenditure, is the institution of private property... With the abolition of private property, the characteristic of human nature which now finds its exercise in this form of emulation, should logically find exercise in other, perhaps nobler and socially more serviceable, activities." Elimination of the KUJ competition, in Veblen's view, would lessen the amount of labor and output required to support the economy. This is similar to what happens upon transition to the fear equilibrium under conditions stated above: output and labor supply fall, individuals enjoy more leisure and are, at least in the short run, happier with less output. As discussed above, the rich may prefer well-protected property rights if inequality is high enough, since moving to the FE would mean losing too much in terms of relative standing. Thus, in a KUJE with relatively high inequality there is likely to be a conflict of interests with regard to institutional quality.

While intuitive in the short run, the negative impact of better institutions on social welfare need not hold for future generations of both dynasties. This is where the second, positive externality comes in. As follows from part 2 of proposition 4, if the spillover effect (α) is strong enough, the long-run KUJE will Pareto dominate the initial FE. The intuition for such reversal of the short-run result is simple. Better institutions make people exert more effort due to constructive envy, which causes productivity growth and raises investment opportunities of future generations. This growth eventually allows everyone to increase relative consumption and decrease effort compared to the initial jump.[43] Note that the long-run Pareto improvement result holds in an equal enough society, as stated in proposition 4. Otherwise, the shift is too costly for the poor dynasty in terms of foregone leisure despite the increase in relative consumption.[44]

To summarize the message of proposition 4, while a strong consumption externality unleashed by better institutions might lead to lower welfare in the short run, a strong spillover effect can more than make up for it in the long run. Such scenario is shown in figure 10.

[43]Clearly, an increase in C_1 and C_2 by the same factor due to rising productivity makes everyone better off since own consumption is valued more than reference consumption ($\theta < 1$).

[44]Note also that in the short run a Pareto improvement is more likely to happen in an unequal society, while in the long run it is the other way round. The reason is that in the former case it is the rich agent who is critical and his preference is to maintain higher inequality, while in the latter case the poor agent is critical and he prefers equality. See the proof of proposition 4 for details.



(a) Welfare of the poor

(b) Welfare of the rich

(c) Effort and relative consumption of the poor

(d) Effort and relative consumption of the rich

Figure 10: From FE to KUJE via better institutions.

5.2 Redistribution

As follows from proposition 1, another way to abandon the fear equilibrium and enable KUJ competition is to make the distribution of endowments more equal. Is there a scope for Pareto-improving redistribution in the short and the long run? The following proposition holds.

Proposition 5. (Welfare and redistribution.) Assume that $k \in [\hat{k}, \tilde{k})$ and the society is in the long-run FE. Then,

I. *Short run* (current generation). $\exists!\ \bar{\bar{\theta}} \in (0,1)$ such that: 1) If $\theta > \bar{\bar{\theta}}$, redistribution shifting the society to a KUJE is never a Pareto improvement; 2) If $\theta \leqslant \bar{\bar{\theta}}$, $\exists!$

$\bar{\bar{k}} \in [\hat{k}, \tilde{k})$, such that Pareto-improving shift to a KUJE is feasible for $k \in [\hat{k}, \bar{\bar{k}}]$. However, the optimal level of inequality from the viewpoint of Agent 2, k^*, always lies in the fear region, that is, $k^* \in [\hat{k}, \tilde{k})$.

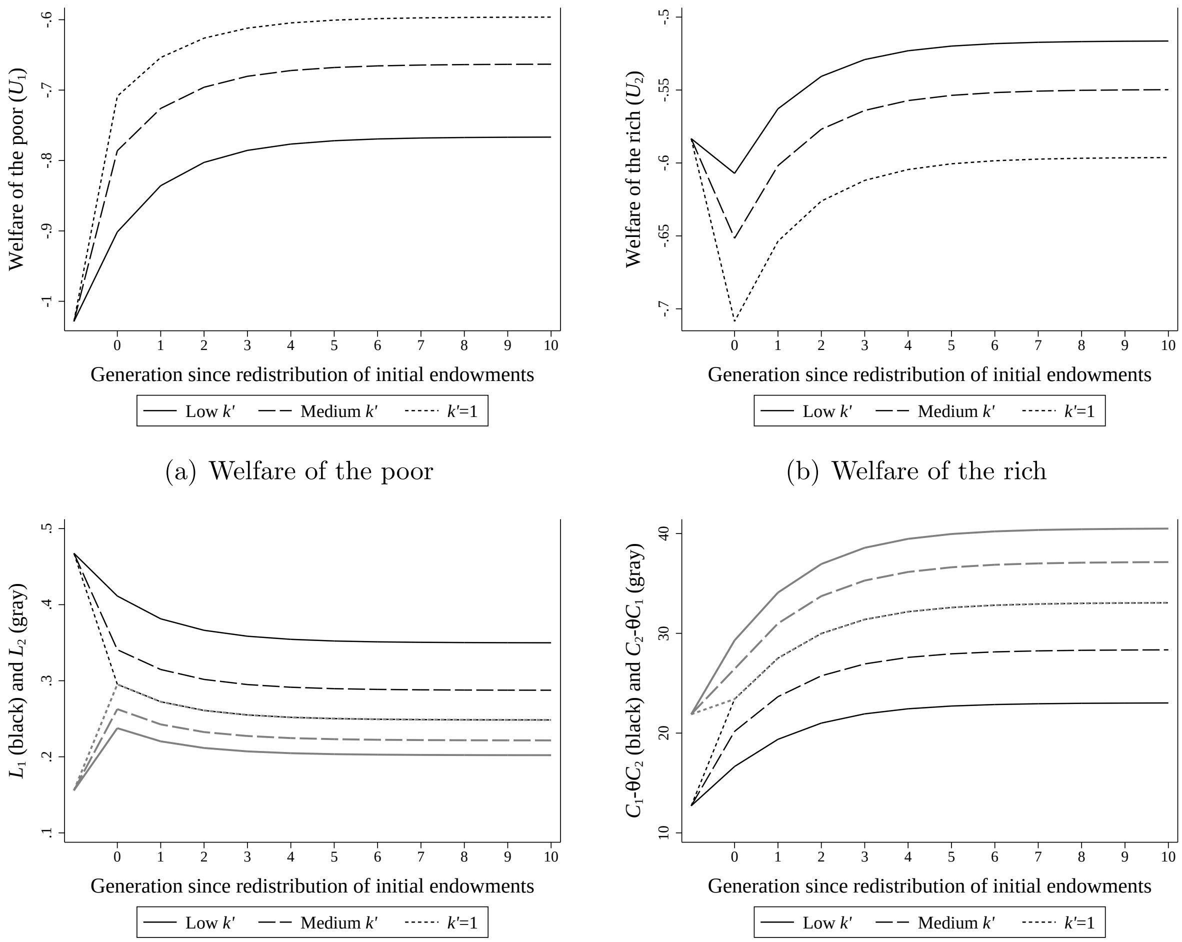
II. *Long run* (future generations). For small enough θ or τ, $\exists!\ \bar{\bar{\alpha}} \in (0,1)$ such that, if $\alpha \geqslant \bar{\bar{\alpha}}$: 1) Pareto-improving redistribution shifting the society to a KUJE is feasible for all $k \in [\hat{k}, \tilde{k}]$; 2) The optimal level of inequality from the viewpoint of the rich dynasty in the long-run KUJE, k^*, is strictly less than 1, that is, $k^* \in [\tilde{k}, 1)$.

Clearly, the poor agent is always in favor of redistribution, both in the short and the long run. It increases the marginal product of his effort while at the same time decreasing that of the rich. From the point of view of the rich agent, redistribution can only be beneficial under certain conditions and only to a certain extent. Specifically, his preferred level of initial inequality is never in the KUJ region in the short run. If comparisons are important (θ is high), it is never optimal to give up the initial wealth advantage and then work hard to compete peacefully with an "enriched" rival. For weak relative standing concerns, however, the boost in own output following the relaxation of the fear constraint might be worth transferring part of the endowment to the poor.

In the long run, redistribution leads to a rise in the effort-saving productivity growth, similar to the effect of better property rights protection. This additional gain raises the welfare of future generations compared to the current one. In fact, if the knowledge spillover effect is strong enough (α is high), Pareto-improving redistribution to the KUJ region is always feasible, regardless of the starting point. The optimal level of initial inequality in the long-run is always in the KUJ region, although it is never a perfect equality. In sum, even if the dynasty of the rich experiences a short-term loss, in the long-run both dynasties can be better off.

Figure 11 shows possible development scenarios for the case in which the rich are worse off in the short run. As figure 11(b) demonstrates, depending on the degree of redistribution (captured by the value of $k' > \tilde{k}$), the rich may be better or worse off in the long run relative to the initial fear-of-envy steady state. Specifically, too much sharing leads to a drastic increase in effort that is not compensated enough by a rise in relative consumption, see figures 11(c) and 11(d). Still, moderate redistribution brings about a Pareto improvement in the long run.

Overall, the results of this section supplement the growth implications of the two sides of envy with welfare analysis. Specifically, as long as the knowledge spillover effect is strong enough, the long-run growth and welfare results are aligned: destructive envy and the fear



(a) Welfare of the poor

(b) Welfare of the rich

(c) Efforts

(d) Relative consumption

Figure 11: From FE to KUJE via redistribution.

of it contribute to stagnation, while constructive envy leads to productivity growth and higher social welfare, as stated in the epigraph to this paper.

6 Concluding remarks

This paper develops a unified framework for the economic analysis of envy by capturing its two main forces, destructive and constructive. The dominant role of envy in the society is determined in equilibrium by the available investment opportunities, the level of fundamental inequality, and the endogenous degree of tolerance for inequality shaped by institutions and culture.

The qualitatively different equilibria that arise in this framework are broadly consistent with evidence on the implications of envy for economic performance and its changing role in the process of development. The "keeping up with the Joneses" equilibrium roughly corresponds to modern advanced economies, in which emulation is an important driver of economic activity. The "fear equilibrium" resembles the adverse effects of envy in developing economies, in which the anticipation of envious retaliation prevents productive investment and retards progress. The different nature of these equilibria yields contrasting comparative statics. In the KUJ equilibrium, envy enhances investment by intensifying emulation, while in the fear equilibrium it reduces output by reinforcing the envy-avoidance behavior. As rising productivity expands investment opportunities, the society experiences an endogenous transition from the fear equilibrium to the KUJ equilibrium causing a qualitative change in the relationship between envy and economic performance.

From a welfare perspective, better institutions and wealth redistribution that move the society away from the low-output fear equilibrium and put it on a KUJ trajectory need not be Pareto improving in the short run, as they unleash the negative consumption externality. In the long run, such policies will increase social welfare due to enhanced productivity growth if the knowledge spillover effect is sufficiently strong.

One important direction for future research is the study of envy-related cultural beliefs and institutions from the perspective of the proposed unified framework. Another interesting subject for further investigation is the interaction between the process of development, inequality, and the endogenous formation of preferences featuring envy.

Appendix

Detailed Form of Lemma 2. The detailed version of equation (12) is:

1. If $AK_2 \geqslant \widehat{A}_{21}$, then

$$C_2^*(C_1) = \begin{cases} AK_2, & \text{if} \quad C_1 \geqslant \widehat{C}_{11}; \\ \theta C_1 + (AK_2)^{1/\sigma}, & \text{if} \quad \widehat{C}_{12} \leqslant C_1 < \widehat{C}_{11}; \\ C_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad \widehat{C}_{13} \leqslant C_1 < \widehat{C}_{12}; \\ C_2^d(C_1), & \text{if} \quad C_1 < \widehat{C}_{13}, \end{cases}$$

where

$$\widehat{C}_{11} \equiv \frac{AK_2 - (AK_2)^{1/\sigma}}{\theta}, \quad \widehat{C}_{12} \equiv \frac{\tau\theta(AK_2)^{1/\sigma}}{1-\tau\theta^2}, \quad \widehat{C}_{13} \equiv \frac{\tau\theta}{1-\tau\theta^2}\left(\frac{1+\theta^2}{2}AK_2\right)^{1/\sigma},$$

and $C_2^d(C_1)$ is implicitly given by

$$C_2 - \theta C_1 = \phi \cdot \left(\frac{C_1 + \theta C_2}{C_2}\right)^{1/\sigma}, \quad \phi \equiv \left(\frac{1+\theta^2}{2\theta(1+\tau)}AK_2\right)^{1/\sigma}. \tag{26}$$

2. If $AK_2 \in [\widehat{A}_{22}, \widehat{A}_{21})$, then

$$C_2^*(C_1) = \begin{cases} AK_2, & \text{if} \quad C_1 \geqslant \widehat{C}_{14}; \\ C_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad \widehat{C}_{13} \leqslant C_1 < \widehat{C}_{14}; \\ C_2^d(C_1), & \text{if} \quad C_1 < \widehat{C}_{13}, \end{cases}$$

where $\widehat{C}_{14} \equiv \tau\theta AK_2$.

3. If $AK_2 \in (\widehat{A}_{23}, \widehat{A}_{22})$, then

$$C_2^*(C_1) = \begin{cases} AK_2, & \text{if} \quad C_1 \geqslant \widehat{C}_{14}; \\ \widetilde{C}_2^d(C_1), & \text{if} \quad \widehat{C}_{15} \leqslant C_1 < \widehat{C}_{14}; \\ C_2^d(C_1), & \text{if} \quad C_1 < \widehat{C}_{15}, \end{cases}$$

where $\widetilde{C}_2^d(C_1)$ is implicitly given by

$$C_1 = \theta C_2 \cdot \left(\frac{1+\tau}{AK_2} \cdot C_2 - 1\right)$$

and $\widehat{C}_{15}$ solves $C_2^d(\widehat{C}_{15}) = \widetilde{C}_2^d(\widehat{C}_{15})$.

The threshold $\widehat{C}_1$ from lemma 2 is defined as $\widehat{C}_1 \equiv \min\{\widehat{C}_{13}, \hat{C}_{14}\}$.

Detailed Form of Lemma 3. The detailed version of equation (15) is:

1. If $AK_1 \geqslant \widehat{A}_{11}$, then

$$C_1^*(C_2) = \begin{cases} \widetilde{C}_1^d(C_2), & \text{if} \quad C_2 \geqslant \widehat{C}_{22}; \\ C_1^d(C_2), & \text{if} \quad \widehat{C}_{21} \leqslant C_2 < \widehat{C}_{22}; \\ \theta C_2 + (AK_1)^{1/\sigma}, & \text{if} \quad C_2 < \widehat{C}_{21}, \end{cases}$$

   where

$$\widetilde{C}_1^d(C_2) \equiv \frac{1+\tau}{\tau} AK_1 - \theta C_2, \quad \widehat{C}_{21} \equiv \frac{(AK_1)^{1/\sigma}}{\theta(\tau-1)},$$

   the function $C_1^d(C_2)$ is implicitly given by

$$C_1 - \theta C_2 = \psi \cdot \left(\frac{C_1}{C_1 + \theta C_2} \right)^{1/\sigma}, \quad \psi \equiv \left(\frac{1+\tau}{\tau} AK_1 \right)^{1/\sigma}, \tag{27}$$

   and $\widehat{C}_{22}$ solves $C_1^d(\widehat{C}_{22}) = \widetilde{C}_1^d(\widehat{C}_{22})$.

2. If $AK_1 \in [\widehat{A}_{12}, \widehat{A}_{11})$, then

$$C_1^*(C_2) = \begin{cases} \widetilde{C}_1^d(C_2), & \text{if} \quad C_2 \geqslant \widehat{C}_{24}; \\ AK_1, & \text{if} \quad \widehat{C}_{23} \leqslant C_2 < \widehat{C}_{24}; \\ \theta C_2 + (AK_1)^{1/\sigma}, & \text{if} \quad C_2 < \widehat{C}_{23}, \end{cases}$$

   where

$$\widehat{C}_{23} \equiv \frac{AK_1 - (AK_1)^{1/\sigma}}{\theta}, \quad \widehat{C}_{24} \equiv \frac{AK_1}{\tau\theta}.$$

3. If $AK_1 < \widehat{A}_{12}$, then

$$C_1^*(C_2) = \begin{cases} \widetilde{C}_1^d(C_2), & \text{if} \quad C_2 \geqslant \widehat{C}_{24}; \\ AK_1, & \text{if} \quad C_2 < \widehat{C}_{24}. \end{cases}$$

The threshold $\widehat{C}_2$ from lemma 3 is defined as $\widehat{C}_2 \equiv \min\{\widehat{C}_{21}, \hat{C}_{24}\}$.

Detailed Form of Proposition 1. The unique subgame perfect equilibrium (C_1^*, C_2^*) of the envy game is determined as follows.

1. If $AK_1 \geqslant \widehat{A}_{11}$ and $AK_2 \geqslant \widehat{A}_{21}$, then:

(a) If $k \geqslant \tilde{k}$, it is the KUJ equilibrium (16);

(b) If $\hat{k} \leqslant k < \tilde{k}$, it is the fear equilibrium (19);

(c) If $k < \hat{k}$, it is the destructive equilibrium implicitly defined by

$$\begin{cases} C_1^* = \min\{C_1^d(C_2^*), \widetilde{C}_1^d(C_2^*)\}; \\ C_2^* = C_2^d(C_1^*). \end{cases}$$

The threshold values of k are given by

$$\tilde{k} \equiv \left[\frac{\theta(\tau-1)}{1-\tau\theta^2}\right]^\sigma, \quad \hat{k} \equiv \tilde{k} \cdot \frac{(1+\theta^2)}{2}. \tag{28}$$

2. If $\widehat{A}_{12} \leqslant AK_1 < \widehat{A}_{11}$ and $AK_2 \geqslant \widehat{A}_{21}$, then:

(a) If $AK_1 \geqslant \widehat{C}_{11}$, it is the full-time KUJ equilibrium (18);

(b) If $AK_1 < \widehat{C}_{11}$ and $m_1(AK_1) \leqslant AK_2 < m_2(AK_1)$, it is the KUJ equilibrium (17), where

$$m_1(AK_1) \equiv \left[\frac{(1-\theta^2)AK_1 - (AK_1)^{1/\sigma}}{\theta}\right]^\sigma, \quad m_2(AK_1) \equiv \left[\frac{(1-\tau\theta^2)AK_1}{\tau\theta}\right]^\sigma;$$

(c) If $AK_2 < m_1(AK_1)$, it is the KUJ equilibrium (16);

(d) If $m_2(AK_1) \leqslant AK_2 < 2m_2(AK_1)/(1+\theta^2)$, it is the fear equilibrium (20);

(e) If $AK_2 \geqslant 2m_2(AK_1)/(1+\theta^2)$, it is the destructive equilibrium in case 2 of (21).

3. If $AK_1 < \widehat{A}_{11}$ and $\widehat{A}_{22} \leqslant AK_2 < \widehat{A}_{21}$, then:

(a) If $k \geqslant \tau\theta$, it is the full-time KUJ equilibrium (18);

(b) If $k < \tau\theta$ and $AK_2 < 2m_2(AK_1)/(1+\theta^2)$, it is the fear equilibrium (20);

(c) If $AK_2 \geqslant 2m_2(AK_1)/(1+\theta^2)$, it is the destructive equilibrium in case 2 of (21).

4. If $AK_1 < \widehat{A}_{11}$ and $\widehat{A}_{23} < AK_2 < \widehat{A}_{22}$, then:

(a) If $k \geqslant \tau\theta$, it is the full-time KUJ equilibrium (18);

(b) If $k < \tau\theta$, it is the destructive equilibrium implicitly defined by

$$\begin{cases} C_1^* = \widetilde{C}_1^d(C_2^*); \\ C_2^* = \min\{C_2^d(C_1^*), \widetilde{C}_2^d(C_1^*)\}. \end{cases}$$

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Supplementary Appendix A

"The Two Sides of Envy"

Journal of Economic Growth

Boris Gershman*

This appendix contains the proofs of all statements from "The Two Sides of Envy."

A Proofs

Proof of Lemma 1. As follows from (8), $d_2^* = 0$ iff $Y_2/Y_1 \geqslant \tau\theta(1-d_1)(1+\tau d_1)$. This clearly holds, if $d_1 = 0$, since $\tau\theta < 1$ and $Y_2 > Y_1$. If $d_1 = (\sqrt{\tau\theta Y_2/Y_1} - 1)/\tau > 0$, the above inequality is equivalent to

$$\sqrt{\frac{Y_2}{Y_1}} \geqslant \frac{\theta(1+\tau)\sqrt{\tau\theta}}{1+\tau\theta^2}.$$

Since $Y_2 > Y_1$, this would always hold if the right-hand side is less or equal than unity, that is, $\theta\sqrt{\tau\theta} \leqslant (1+\tau\theta^2)/(1+\tau)$. Since $\tau < 1/\theta$, $\sup\{\theta\sqrt{\tau\theta}\} = \inf\{(1+\tau\theta^2)/(1+\tau)\} = \theta$. Hence, $\theta\sqrt{\tau\theta} \leqslant (1+\tau\theta^2)/(1+\tau)$, given that $\tau\theta < 1$ and $Y_2 > Y_1$, and lemma 1 gives the unique second-stage equilibrium.

Proof of Lemma 2. Agent 2 is solving (11) subject to (9). First, consider the case $Y_2 \leqslant Y_1/\tau\theta$, in which $d_1^* = 0$ and

$$U_2 = \frac{(Y_2 - \theta Y_1)^{1-\sigma}}{1-\sigma} - \frac{Y_2}{AK_2}$$

is strictly concave in Y_2. Since $Y_2 \leqslant AK_2$, there are two subcases.

1) If $Y_1 \leqslant \tau\theta AK_2$, the first-order conditions yield

$$Y_2 = \begin{cases} \theta Y_1 + (AK_2)^{1/\sigma}, & \text{if} \quad Y_1 \geqslant \widehat{C}_{12}; \\ Y_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad Y_1 < \widehat{C}_{12}. \end{cases} \tag{A.1}$$

Since in this subcase $Y_1 \leqslant \tau\theta AK_2$, for $AK_2 \geqslant \widehat{A}_{21}$ the best response of Agent 2 is given by (A.1), while for $AK_2 < \widehat{A}_{21}$ it is just $Y_2 = Y_1/\tau\theta$.

2) If $Y_1 > \tau\theta AK_2$, the first-order conditions yield

$$Y_2 = \begin{cases} AK_2, & \text{if} \quad Y_1 \geqslant \widehat{C}_{11}; \\ \theta Y_1 + (AK_2)^{1/\sigma}, & \text{if} \quad Y_1 < \widehat{C}_{11}. \end{cases} \tag{A.2}$$

Since in this subcase $Y_1 > \tau\theta AK_2$, for $AK_2 \geqslant \widehat{A}_{21}$ the best response of Agent 2 is given by (A.2), while for $AK_2 < \widehat{A}_{21}$ it is just $Y_2 = AK_2$.

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Putting the two subcases together we obtain the best-response function for the case $Y_2 \leqslant Y_1/\tau\theta$:

$$Y_2 = \left[\begin{array}{ll} \begin{cases} AK_2, & \text{if} \quad Y_1 \geqslant \widehat{C}_{11}; \\ \theta Y_1 + (AK_2)^{1/\sigma}, & \text{if} \quad \widehat{C}_{12} \leqslant Y_1 < \widehat{C}_{11}; \\ Y_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad Y_1 < \widehat{C}_{12}, \end{cases} & \text{for} \quad AK_2 \geqslant \widehat{A}_{21}; \\ \begin{cases} AK_2, & \text{if} \quad Y_1 \geqslant \widehat{C}_{14}; \\ Y_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad Y_1 < \widehat{C}_{14}, \end{cases} & \text{for} \quad AK_2 < \widehat{A}_{21}. \end{array} \right. \tag{A.3}$$

Now consider the destructive case, $Y_2 > Y_1/\tau\theta$, in which

$$U_2 = \frac{1}{1-\sigma} \cdot \left(\sqrt{\frac{Y_1 Y_2}{\tau\theta}} \cdot (1+\theta^2) - \frac{\theta(\tau+1)}{\tau} Y_1 \right)^{1-\sigma} - \frac{Y_2}{AK_2}$$

is strictly concave in Y_2. Since $Y_2 \leqslant AK_2$, the only relevant case to consider is when $Y_1 < \tau\theta AK_2$. The interior optimum, $Y_2^d(Y_1)$, is uniquely defined by the first-order condition

$$\omega_2(Y_1, Y_2) \equiv \left(\sqrt{\frac{Y_1 Y_2}{\tau\theta}} \cdot (1+\theta^2) - \frac{\theta(\tau+1)}{\tau} Y_1 \right)^{-\sigma} \cdot \left(\frac{1+\theta^2}{2\sqrt{\tau\theta}} \cdot \sqrt{\frac{Y_1}{Y_2}} \right) - \frac{1}{AK_2} = 0. \tag{A.4}$$

The right derivative of U_2 at point $Y_2 = Y_1/\tau\theta$ is positive (negative) iff $Y_1 < \widehat{C}_{13}$ ($Y_1 > \widehat{C}_{13}$). Furthermore, $\widehat{C}_{13} < \tau\theta AK_2$ iff $AK_2 > \widehat{A}_{22}$, and $\widehat{C}_{13} < \widehat{C}_{12}$, since $\theta \in (0,1)$.

Next, consider the right derivative of U_2 at point $Y_2 = AK_2$, that is, $\omega_2(Y_1, AK_2)$. Equation (9) implies that for $Y_2 = AK_2$ the minimal Y_1 that can guarantee that $d_1^* \leqslant 1$ is equal to $\bar{Y}_1 \equiv \tau\theta AK_2/(1+\tau)^2$ which corresponds to $C_1 = 0$. Since consumption of Agent 1 will always be positive in equilibrium, we focus on the analysis of $\omega_2(Y_1, AK_2)$ for $Y_1 \in [\bar{Y}_1, \tau\theta AK_2]$. It is straightforward to show that: a) $\omega_2(\bar{Y}_1, AK_2) < 0$ iff $AK_2 > \widehat{A}_{23}$; b) $\omega_2'(Y_1^*, AK_2) = 0$, where

$$Y_1^* \equiv \left[\frac{\sigma - 1}{\sigma - 1/2} \cdot \frac{1+\theta^2}{2\theta^2(1+\tau)} \right]^2 \cdot \tau\theta AK_2;$$

c) $\omega_2(\tau\theta AK_2, AK_2) > 0$ iff $AK_2 < \widehat{A}_{22}$. It follows from these properties and the intermediate value theorem that on the segment $[\bar{Y}_1, \tau\theta AK_2]$ the function $\omega_2(Y_1, AK_2)$ has a unique root $\widehat{Y}_{15}$, if $AK_2 \leqslant \widehat{A}_{22}$, and no roots, if $AK_2 > \widehat{A}_{22}$, in which case $\omega_2(Y_1, AK_2)$ is negative on the whole segment.

The preceding analysis implies that for the case $Y_2 > Y_1/\tau\theta$ the best response function is

$$Y_2 = \left[\begin{array}{ll} \begin{cases} Y_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad \widehat{C}_{13} \leqslant Y_1 < \tau\theta AK_2; \\ Y_2^d(Y_1), & \text{if} \quad \bar{Y}_1 < Y_1 < \widehat{C}_{13}, \end{cases} & \text{for} \quad AK_2 \geqslant \widehat{A}_{22}; \\ \begin{cases} AK_2, & \text{if} \quad \widehat{Y}_{15} \leqslant Y_1 < \tau\theta AK_2; \\ Y_2^d(Y_1), & \text{if} \quad \bar{Y}_1 < Y_1 < \widehat{Y}_{15}, \end{cases} & \text{for} \quad \widehat{A}_{23} < AK_2 < \widehat{A}_{22}. \end{array} \right. \tag{A.5}$$

To express (A.5) in terms of consumption, as stated in lemma 2, note that if destruction takes place

$$\begin{cases} C_1 = (1-d_1^*) Y_1 = \dfrac{1+\tau}{\tau} Y_1 - \sqrt{\dfrac{\theta}{\tau} Y_1 Y_2}; \\ C_2 = p_2^* Y_2 = \sqrt{\dfrac{Y_1 Y_2}{\tau\theta}}. \end{cases} \implies \begin{cases} Y_1 = \dfrac{\tau}{1+\tau}(C_1 + \theta C_2); \\ Y_2 = \theta(1+\tau) \cdot \dfrac{C_2^2}{C_1 + \theta C_2}. \end{cases} \tag{A.6}$$

Substitution into (A.4) yields the expression for $C_2^d(C_1)$. Take the implicit derivative to get

$$\frac{dC_2}{dC_1} = \frac{\theta + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2}}{1 + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{C_1}{C_2}} > 0, \tag{A.7}$$

since the optimal response requires $C_2 > \theta C_1$. The latter also implies that $dC_2/dC_1 < C_2/C_1$. Finally,

$$\frac{d^2C_2}{dC_1^2} = \left(\sigma + \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{C_1}{C_2}\right)^{-1} \cdot \frac{C_2 - C_1 \cdot \frac{dC_2}{dC_1}}{C_2(C_1 + \theta C_2)} \cdot \left[\frac{(1+\theta^2)\left(C_1 \frac{dC_2}{dC_1} - C_2\right)}{C_1 + \theta C_2} - \frac{\frac{dC_2}{dC_1}(C_2 - \theta C_1)}{C_2}\right] < 0,$$

since $C_2 > \theta C_1$ and $0 < dC_2/dC_1 < C_2/C_1$. Hence, $C_2^d(C_1)$ is strictly increasing and concave. For the case when $Y_2 = AK_2$ and $AK_2 > Y_1/\tau\theta$ the second equation of the system (A.6) immediately yields the implicit equation for $\widetilde{C}_2^d(C_1)$. The value $\widehat{Y}_{15}$ translates into $\widehat{C}_{15}$, such that $C_2^d(\widehat{C}_{15}) = \widetilde{C}_2^d(\widehat{C}_{15})$.

Putting together the peaceful and destructive cases, (A.3) and (A.5), expressed in consumption terms, yields the detailed form of lemma 2 in the Appendix. Note that Agent 2 is always able to guarantee positive relative consumption and thus, avoid negatively infinite utility. In particular, choosing the feasible $Y_2 = AK_2$ in the peaceful case yields the relative consumption of at least $AK_2 - \theta AK_1$, which is positive, since $K_2 > K_1$ and $\theta < 1$. In the destructive case, if $Y_2 = AK_2$ and $Y_1 = AK_1$, relative consumption is positive iff $k < \tau\theta(1+1/\theta^2)^2/(\tau+1)^2$ which always holds, since the right-hand side of the latter inequality is always greater than one under the maintained assumption $\tau\theta < 1$.

Proof of Lemma 3. Agent 1 is solving (13) subject to (9). In the peaceful case, $Y_1 \geqslant \tau\theta Y_2$,

$$U_1 = \frac{(Y_1 - \theta Y_2)^{1-\sigma}}{1-\sigma} - \frac{Y_1}{AK_1}$$

is strictly concave in Y_1. Since $Y_1 \leqslant AK_1$, the only relevant case to consider is when $Y_2 < AK_1/\tau\theta$. First, note that the right derivative of U_1 at point $Y_1 = \tau\theta Y_2$ is positive (negative) iff $Y_2 < \widehat{C}_{21}$ ($Y_2 > \widehat{C}_{21}$), while the left derivative of U_1 at point $Y_1 = AK_1$ is positive (negative) iff $Y_2 < \widehat{C}_{23}$ ($Y_2 > \widehat{C}_{23}$). Second, observe that

$$\widehat{C}_{21} < \widehat{C}_{23} \iff \widehat{C}_{21} < AK_1/\tau\theta \iff \widehat{C}_{23} > AK_1/\tau\theta \iff AK_1 > \widehat{A}_{11}.$$

Together these properties imply that for the case $Y_1 \geqslant \tau\theta Y_2$ the best-response function is

$$Y_1 = \left[\begin{array}{lll} \begin{cases} \tau\theta Y_2, & \text{if } \widehat{C}_{21} \leqslant Y_2 < AK_1/\tau\theta; \\ \theta Y_2 + (AK_1)^{1/\sigma}, & \text{if } Y_2 < \widehat{C}_{21}, \end{cases} & \text{for } AK_1 \geqslant \widehat{A}_{11}; \\ \begin{cases} AK_1, & \text{if } \widehat{C}_{23} \leqslant Y_2 < AK_1/\tau\theta; \\ \theta Y_2 + (AK_1)^{1/\sigma}, & \text{if } Y_2 < \widehat{C}_{23}, \end{cases} & \text{for } \widehat{A}_{12} \leqslant AK_1 < \widehat{A}_{11}; \\ AK_1, & \text{for } AK_1 < \widehat{A}_{12}. \end{array}\right. \tag{A.8}$$

Next, consider the destructive case $Y_1 < \tau\theta Y_2$, in which

$$U_1 = \frac{1}{1-\sigma} \cdot \left(\frac{1+\tau}{\tau} Y_1 - 2\sqrt{\frac{\theta}{\tau} Y_1 Y_2}\right)^{1-\sigma} - \frac{Y_1}{AK_1}.$$

Assumption $\sigma > 1$ is sufficient for U_1 to be strictly concave in Y_1. In particular, simple differentiation shows that the sign of d^2U_1/dY_1^2 coincides with the sign of $h(r) \equiv -(1+\sigma)r^2 + \xi(2\sigma + 1/2)r - \sigma\xi^2$, where

$r \equiv \sqrt{\theta Y_2/\tau Y_1}$ and $\xi \equiv (\tau+1)/\tau$. It is easy to show that the maximum value of h is proportional to $1-8\sigma<0$ and so, U_1 is concave under $\sigma>1$. The interior optimum, $Y_1^d(Y_2)$, is given by the first-order condition:

$$\omega_1(Y_2,Y_1) \equiv \left(\frac{1+\tau}{\tau}Y_1 - 2\sqrt{\frac{\theta}{\tau}Y_1Y_2}\right)^{-\sigma}\cdot\left(\frac{1+\tau}{\tau} - \sqrt{\frac{\theta}{\tau}\cdot\frac{Y_2}{Y_1}}\right) - \frac{1}{AK_1} = 0. \tag{A.9}$$

Since $Y_1 \leqslant AK_1$ there are two subcases.

1) Consider the subcase $Y_2 > AK_1/\tau\theta$. Consider the left derivative of U_1 at point $Y_1 = AK_1$, that is, $\omega_1(Y_2, AK_1)$. Under assumption (10) the following properties hold: a) $\omega_1(AK_1/\tau\theta, AK_1) > 0$ iff $AK_1 < \widehat{A}_{11}$; b) $\omega_1'(Y_2, AK_1) > 0$ for $Y_2 > AK_1/\tau\theta$; c) $\lim_{Y_2\to\widetilde{Y}_2}\omega_1(Y_2, AK_1) = +\infty$, where $\widetilde{Y}_2 \equiv (1+\tau)^2AK_1/4\tau\theta$. Hence, by the intermediate value theorem, for $AK_1 \geqslant \widehat{A}_{11}$ there exists a unique value $\widehat{Y}_{22}$ such that $\omega_1(\widehat{Y}_{22}, AK_1) = 0$. Altogether for this subcase the best response of Agent 1 is $Y_1 = AK_1$ for $AK_1 < \widehat{A}_{11}$, while for $AK_1 \geqslant \widehat{A}_{11}$ it is $Y_1 = AK_1$, if $Y_2 \geqslant \widehat{Y}_{22}$, and $Y_1^d(Y_2)$, otherwise.

2) Now consider the subcase $Y_2 \leqslant AK_1/\tau\theta$. Note that the right derivative of U_1 at point $Y_1 = \tau\theta Y_2$ is positive iff $Y_2 < \widehat{C}_{21}$. Hence, for $AK_1 \geqslant \widehat{A}_{11}$ the best response is $Y_1 = \tau\theta Y_2$, if $Y_2 < \widehat{C}_{21}$, and $Y_1^d(Y_2)$, otherwise. For $AK_1 < \widehat{A}_{11}$ the best response is $Y_1 = \tau\theta Y_2$ for any $Y_2 \leqslant AK_1/\tau\theta$.

Putting the two subcases together we obtain the best-response function for the case $Y_1 < \tau\theta Y_2$:

$$Y_1 = \left[\begin{array}{ll} \begin{cases} AK_1, & \text{if} \quad Y_2 \geqslant \widehat{Y}_{22}; \\ Y_1^d(Y_2), & \text{if} \quad \widehat{C}_{21} \leqslant Y_2 < \widehat{Y}_{22}; \\ \tau\theta Y_2, & \text{if} \quad Y_2 < \widehat{C}_{21}, \end{cases} & \text{for} \quad AK_1 \geqslant \widehat{A}_{11}; \\ \begin{cases} AK_1, & \text{if} \quad Y_2 \geqslant AK_1/\tau\theta; \\ \tau\theta Y_2, & \text{if} \quad Y_2 < AK_1/\tau\theta, \end{cases} & \text{for} \quad AK_1 < \widehat{A}_{11}. \end{array}\right. \tag{A.10}$$

Substitute of (A.6) into (A.9) yields the expression for $C_1^d(C_2)$. Take the implicit derivative to get

$$\frac{dC_1}{dC_2} = \frac{\theta - \frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}}{1-\frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}\cdot\frac{C_2}{C_1}} > 0, \tag{A.11}$$

since $\sigma > 1$ and the optimal response requires $C_1 > \theta C_2$. The latter also implies that $dC_1/dC_2 < C_1/C_2$. Finally,

$$\frac{d^2C_1}{dC_2^2} = \frac{\theta}{\sigma}\left(1-\frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}\cdot\frac{C_2}{C_1}\right)^{-1}\left[\frac{\theta\left(\frac{dC_1}{dC_2}\cdot\frac{C_2}{C_1}-1\right)^2}{C_1+\theta C_2} + \frac{(C_1-\theta C_2)\left(\frac{dC_1}{dC_2}+\theta\right)\left(1-\frac{dC_1}{dC_2}\cdot\frac{C_2}{C_1}\right)}{(C_1+\theta C_2)^2}\right] > 0,$$

since $\sigma > 1$, $C_1 > \theta C_2$, and $dC_1/dC_2 < C_1/C_2$. Hence, $C_1^d(C_2)$ is strictly increasing and convex. For the case when $Y_1 = AK_1$ and $AK_1 < \tau\theta Y_2$ the top equation in (A.6) immediately yields the expression for $\widetilde{C}_1^d(C_2)$. The value for $\widehat{Y}_{22}$ translates into $\widehat{C}_{22}$, such that $C_1^d(\widehat{C}_{22}) = \widetilde{C}_1^d(\widehat{C}_{22})$. Putting together the peaceful and destructive cases, (A.8) and (A.10), expressed in consumption terms, yields the detailed statement of lemma 3 in the Appendix. Note that Agent 1 is always able to guarantee positive relative consumption and thus, avoid negatively infinite utility. In particular, choosing the feasible $Y_1 = AK_1$ in the peaceful case yields the relative consumption of at least $AK_1 - \theta AK_2$ which is positive, since the condition for the peaceful case $K_1 > \tau\theta K_2$ and $\tau > 1$ imply that $K_1 > \theta K_2$. In the destructive case, if $Y_1 = AK_1$ and $Y_2 = AK_2$, relative consumption is positive iff $k > 4\tau\theta/(1+\tau)^2$ which holds by assumption (10).

Proof of Proposition 1. This proof derives the conditions for all feasible types of equilibria outlined in the detailed form of proposition 1 in the Appendix. Derivations rely of the detailed forms of lemmas 2 and 3 stated in the Appendix. It is straightforward to show that no other equilibrium configurations are possible in all of the cases considered below, since the necessary restrictions that must hold are inconsistent with each other.

In case 1, $AK_1 \geqslant \widehat{A}_{11}$ and $AK_2 \geqslant \widehat{A}_{21}$. Note that the latter condition holds automatically, since $K_2 > K_1$ and $\widehat{A}_{11} > \widehat{A}_{21}$, given that $\tau\theta < 1$. From lemmas 2 and 3:

(a) A pair (16) is an equilibrium if $C_1^* \geqslant \widehat{C}_{12}$, $C_1^* < \widehat{C}_{11}$, and $C_2^* < \widehat{C}_{21}$. The first and the third conditions are equivalent to $k \geqslant \tilde{k}$, while the second inequality holds automatically since $K_2 > K_1$, $AK_1 > \widehat{A}_{11}$, and $\tau\theta < 1$.

(b) A pair (19) is an equilibrium if $C_1^* \geqslant \widehat{C}_{13}$, $C_1^* < \widehat{C}_{12}$, and $C_2^* \leqslant \widehat{C}_{21}$. The first two conditions yield $\hat{k} \leqslant k < \tilde{k}$, while the third holds as equality.

(c) The borderline fear equilibrium happens when $\tau\theta\widehat{C}_{21} = \widehat{C}_{13}$, that is, when $k = \hat{k}$. If $\tau\theta\widehat{C}_{21} < \widehat{C}_{13}$, that is, $k < \hat{k}$, the only possibility is the destructive equilibrium of part 1(c). To prove existence, note, first, that $C_2^d(0) = \phi \cdot \theta^{1/\sigma}$ and $\widetilde{C}_1^d(C_2) = 0$ iff $C_2 = \bar{C}_2 \equiv (1+\tau)AK_1/\tau\theta$. Next, assumption $k > \underline{k}$ implies that $\bar{C}_2 > 4 \cdot AK_2/(1+\tau)$. Hence, a sufficient condition for $\bar{C}_2 > C_2^d(0)$ would be

$$(AK_2)^{\frac{\sigma-1}{\sigma}} > (1+\tau)^{\frac{\sigma-1}{\sigma}} \cdot \frac{1}{4} \cdot \left(\frac{1+\theta^2}{2}\right)^{1/\sigma}.$$

It follows from the condition $AK_2 \geqslant \widehat{A}_{21}$ and the assumption $\widehat{A}_{22} > \widehat{A}_{23}$ that

$$(AK_2)^{\frac{\sigma-1}{\sigma}} > (1+\tau)^{\frac{\sigma-1}{\sigma}} \cdot \left(\frac{1+\theta^2}{2}\right)^{1/\sigma} > (1+\tau)^{\frac{\sigma-1}{\sigma}} \cdot \frac{1}{4} \cdot \left(\frac{1+\theta^2}{2}\right)^{1/\sigma}$$

implying that $\bar{C}_2 > C_2^d(0)$. This guarantees existence. Uniqueness follows from the properties of best responses. Specifically, as shown in lemmas 2 and 3, $C_2^d(C_1)$ is strictly increasing and concave while $C_1^d(C_2)$ is strictly increasing and convex. Furthermore, direct comparison of (A.7) and (A.11) shows that the slope of the inverse of C_1^d is always steeper than that of C_2^d, which ensures single crossing.

Now consider case 2, in which $\widehat{A}_{12} \leqslant AK_1 < \widehat{A}_{11}$ and $AK_2 \geqslant \widehat{A}_{21}$. From lemmas 2 and 3:

(a) A pair (18) is an equilibrium if $C_1^* = AK_1 \geqslant \widehat{C}_{11}$, $C_2^* = AK_2 \geqslant \widehat{C}_{23}$, and $C_2^* < \widehat{C}_{24}$. Since $K_1 < K_2$, $\widehat{C}_{23} < \widehat{C}_{11}$ and the first condition implies the second one. The third condition yields $k > \tau\theta$ which is implied by $AK_1 \geqslant \widehat{C}_{11}$ taken together with $AK_2 \geqslant \widehat{A}_{21}$. Hence, $AK_1 \geqslant \widehat{C}_{11}$ is the only relevant condition.

(b) A pair (17) is an equilibrium if $\widehat{C}_{12} \leqslant C_1^* < \widehat{C}_{11}$ and $\widehat{C}_{23} \leqslant C_2^* \leqslant \widehat{C}_{24}$ which yields the conditions stated in the detailed form of proposition 1, part 2(b). Note that conditions $\widehat{C}_{12} \leqslant C_1^*$ and $C_2^* \leqslant \widehat{C}_{24}$ are equivalent. Note also that in the relevant region ($k < 1$) the expression for $m_1(AK_1)$ is always well-defined.

(c) A pair (16) is an equilibrium if $C_1^* \geqslant \widehat{C}_{12}$, $C_1^* < \widehat{C}_{11}$, and $C_2^* < \widehat{C}_{23}$. The third condition implies the second since $K_2 > K_1$. Next, the first inequality is equivalent to $k \geqslant \tilde{k}$ and is redundant since it is implied by the third inequality and the condition $AK_1 \leqslant \widehat{A}_{11}$ taken together.

(d) A pair (20) is an equilibrium if $\widehat{C}_{13} \leqslant C_1^* < \widehat{C}_{12}$, $C_2^* \leqslant \widehat{C}_{24}$, and $C_2^* \geqslant \widehat{C}_{23}$. The first two inequalities yield the conditions stated in the detailed form of proposition 1, part 2(d). The third condition holds as equality, while the last one is equivalent to $AK_1 \leqslant \widehat{A}_{11}$ presumed to hold for this case.

(e) The borderline fear equilibrium happens when $AK_1 = \widehat{C}_{13}$. If $AK_1 < \widehat{C}_{13}$, the only possibility is the destructive equilibrium from part 2(e). For the proof of existence see case 1(c); uniqueness follows immediately from the properties of $\widetilde{C}_1^d(C_2)$ and $C_2^d(C_1)$.

Consider case 3, in which $AK_1 < \widehat{A}_{11}$ and $\widehat{A}_{22} \leqslant AK_2 < \widehat{A}_{21}$. Technically, this case covers two subcases depending on how AK_1 compares to $\widehat{A}_{12}$. However, they yield the same results. Note also that the subregion in which $AK_1 < \widehat{A}_{12}$ exists iff $\widehat{A}_{12} > \underline{k} \cdot \widehat{A}_{22}$. From lemmas 2 and 3:

(a) If $AK_1 \geqslant \widehat{A}_{12}$, a pair (18) is an equilibrium if $C_1^* \geqslant \widehat{C}_{14}$, $C_2^* \geqslant \widehat{C}_{23}$, and $C_2^* \leqslant \widehat{C}_{24}$. The first and the third inequalities are equivalent to $k \geqslant \tau\theta$, while the second inequality is implied by $K_2 > K_1$ together with the conditions $AK_2 < \widehat{A}_{21}$ and $\tau\theta < 1$. If $AK_1 < \widehat{A}_{12}$, the only two conditions are $C_1^* \geqslant \widehat{C}_{14}$ and $C_2^* < \widehat{C}_{24}$ yielding again $k \geqslant \tau\theta$.

(b) If $AK_1 \geqslant \widehat{A}_{12}$, a pair (20) is an equilibrium if $\widehat{C}_{13} \leqslant C_1^* < \widehat{C}_{14}$, $C_2^* \geqslant \widehat{C}_{23}$ and $C_2^* \leqslant \widehat{C}_{24}$. The last inequality holds as equality, while the third one is implied by $AK_1 < \widehat{A}_{11}$. The first two conditions yield those stated in the detailed form of proposition 1, part 3(b). If $AK_1 < \widehat{A}_{12}$, the relevant conditions are $\widehat{C}_{13} \leqslant C_1^* < \widehat{C}_{14}$ and $C_2^* \leqslant \widehat{C}_{24}$ yielding the same set of restrictions as for $AK_1 \geqslant \widehat{A}_{12}$.

(c) See the proof for case 2(e) above.

Finally, consider case 4, in which $AK_1 < \widehat{A}_{11}$ and $\widehat{A}_{23} < AK_2 < \widehat{A}_{22}$. Technically, this case covers two subcases depending on how AK_1 compares to $\widehat{A}_{12}$. However, they yield the same results. Note also that the subregion in which $AK_1 < \widehat{A}_{12}$ exists iff $\widehat{A}_{12} > \underline{k} \cdot \widehat{A}_{23}$, and the other subregion only exists iff $\widehat{A}_{12} < \widehat{A}_{22}$. From lemmas 2 and 3:

(a) See the proof for case 3(a) above.

(b) The borderline case happens when $k = \tau\theta$. If $k < \tau\theta$, the only possibility is the destructive equilibrium from part 4(b). For the proof of existence see case 1(c); uniqueness follows immediately from the properties of $\widetilde{C}_1^d(C_2)$, $C_2^d(C_1)$, and $\widetilde{C}_2^d(C_1)$.

Proof of Proposition 2. The results for cases (16), (19), (18) and (20) follow immediately from the expressions for individual outputs. For case (17) the effects of θ, τ, and A are straightforward, while for λ we have $\partial C_1/\partial\lambda = AK > 0$ and

$$\frac{\partial C_2}{\partial \lambda} = \left(\theta - \frac{1}{\sigma}(AK_2)^{\frac{1-\sigma}{\sigma}}\right) AK$$

which is positive iff $AK_2 > (\sigma\theta)^{\frac{\sigma}{1-\sigma}}$. It follows that $\partial C/\partial\lambda > 0$ iff $AK_2 > [\sigma(1+\theta)]^{\frac{\sigma}{1-\sigma}}$ which always holds since for this case $AK_2 \geqslant \widehat{A}_{21} > 1$ and $[\sigma(1+\theta)]^{\frac{\sigma}{1-\sigma}} < 1$.

For the first case in (21), consider the system defining the equilibrium:

$$\begin{cases} f_1 \equiv C_1 - \theta C_2 - \psi[C_1/(C_1+\theta C_2)]^{1/\sigma} = 0; \\ f_2 \equiv C_2 - \theta C_1 - \phi[(C_1+\theta C_2)/C_2]^{1/\sigma} = 0. \end{cases}$$

Using the equilibrium conditions, it can be shown that

$$[D_{\mathbf{C}}\mathbf{f}]^{-1} = \frac{1}{\Delta_1}\left[\begin{array}{ll} 1+\frac{1}{\sigma}\cdot\frac{C_2-\theta C_1}{C_1+\theta C_2}\cdot\frac{C_1}{C_2} & \theta-\frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2} \\ \theta+\frac{1}{\sigma}\cdot\frac{C_2-\theta C_1}{C_1+\theta C_2} & 1-\frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}\cdot\frac{C_2}{C_1} \end{array}\right],$$

where $\mathbf{C}$ is the vector of consumption levels, $\mathbf{f}$ is the vector of f_1 and f_2, and

$$\Delta_1 \equiv \det(D_{\mathbf{C}}\mathbf{f}) = 1-\theta^2+\frac{(C_1-\theta C_2)^2(C_2-\theta C_1)}{\sigma(C_1+\theta C_2)C_1C_2} > 0.$$

It follows from this expression that all elements of $[D_{\mathbf{C}}\mathbf{f}]^{-1}$ are positive since $\sigma > 1$ and $\theta C_1 < C_2 < C_1/\theta$ in equilibrium.

For the effects of λ, note that, by the implicit function theorem, $D_\lambda\mathbf{C} = -[D_{\mathbf{C}}\mathbf{f}]^{-1}\cdot D_\lambda\mathbf{f}$ and

$$D_\lambda\mathbf{f} = -\left[\begin{array}{c} \frac{\psi_\lambda}{\psi}(C_1-\theta C_2) \\ \frac{\phi_\lambda}{\phi}(C_2-\theta C_1) \end{array}\right] = \frac{1}{\sigma}\left[\begin{array}{c} -\frac{1}{\lambda}(C_1-\theta C_2) \\ \frac{1}{1-\lambda}(C_2-\theta C_1) \end{array}\right].$$

It follows that the sign of $\partial C_1/\partial\lambda$ coincides with the sign of

$$\left[1+\frac{1}{\sigma}\cdot\frac{C_2-\theta C_1}{C_1+\theta C_2}\cdot\frac{C_1}{C_2}\right]\cdot(C_1-\theta C_2)-\frac{\theta\lambda}{1-\lambda}\cdot\left[1-\frac{1}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}\right]\cdot(C_2-\theta C_1).$$

Dividing (26) by (27) and denoting $x \equiv C_1/C_2 \in (\theta, 1)$, in equilibrium we have

$$\frac{\lambda}{1-\lambda} = \left(\frac{x-\theta}{1-\theta x}\right)^\sigma\cdot\frac{\tau(1+\theta^2)}{2\theta(1+\tau)^2}\cdot\frac{(x+\theta)^2}{x}.$$

Plugging this expression for $\lambda/(1-\lambda)$ in the previous equation and making transformations, we get

$$\frac{\sigma(\theta+x)+x(1-\theta x)}{\sigma(\theta+x)-(x-\theta)}-\frac{\tau(1+\theta^2)}{2(1+\tau)^2}\cdot\left(\frac{x-\theta}{1-\theta x}\right)^{\sigma-1}\cdot\frac{(x+\theta)^2}{x}.$$

The first term is always greater than 1. The second term is increasing in x and at $x=1$ simplifies to $[\tau(1+\theta^2)(1+\theta)^2]/[2(1+\tau)^2]$. Since $\tau > 1$, the supremum of the latter expression is equal to $[(1+\theta^2)(1+\theta)^2/4] \leqslant 1$ for all $\theta \in (0,1)$. Hence, the second term is always less than 1 and $\partial C_1/\partial\lambda > 0$.

Although the sign of $\partial C_2/\partial\lambda$ is ambiguous, the sign of $\partial C/\partial\lambda$ is not, as it coincides with that of

$$\frac{\sigma(1+\theta)(x+\theta)+(1-\theta x)(1+x)}{\sigma(1+\theta)(x+\theta)-\theta(x-\theta)(1+1/x)}-\frac{\tau(1+\theta^2)}{2(1+\tau)^2}\cdot\left(\frac{x-\theta}{1-\theta x}\right)^{\sigma-1}\cdot\frac{(x+\theta)^2}{x}.$$

As above, the first term of this expression is always greater than 1, while the second is always less than 1. Hence, $\partial C/\partial\lambda > 0$.

For the effects of θ, note that $D_\theta\mathbf{C} = -[D_{\mathbf{C}}\mathbf{f}]^{-1}\cdot D_\theta\mathbf{f}$ and

$$D_\theta\mathbf{f} = \left[\begin{array}{c} C_2\cdot\left(\frac{1}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}-1\right) \\ -C_1+\frac{1}{\sigma}\cdot\frac{C_2-\theta C_1}{C_1+\theta C_2}\cdot\frac{(1-\theta^2)C_1-2\theta^3C_2}{\theta(1+\theta^2)} \end{array}\right].$$

It follows that the sign of $\partial C_1/\partial\theta$ is determined by the sign of

$$C_2+\theta C_1-\frac{1}{\sigma}\frac{C_2-\theta C_1}{C_1+\theta C_2}\cdot\left(\frac{(1-\theta^2)C_1-2\theta^3C_2}{1+\theta^2}-C_1\right).$$

Since $(1-\theta^2)C_1 - 2\theta^3 C_2 - (1+\theta^2)C_1 = -2\theta^2 C_1 - 2\theta^3 C_2 < 0$, it is clear that $\partial C_1/\partial\theta > 0$. As discussed in the main text, the sign of $\partial C_2/\partial\theta$ is generally ambiguous.

For the effects of τ and A, note that $D_\tau \mathbf{C} = -[D_\mathbf{C}\mathbf{f}]^{-1} \cdot D_\tau \mathbf{f}$, $D_A \mathbf{C} = -[D_A \mathbf{f}]^{-1} \cdot D_A \mathbf{f}$, where

$$D_\tau \mathbf{f} = -\begin{bmatrix} \frac{\psi_\tau}{\psi}(C_1 - \theta C_2) \\ \frac{\phi_\tau}{\phi}(C_2 - \theta C_1) \end{bmatrix}, \qquad D_A \mathbf{f} = -\begin{bmatrix} \frac{\psi_A}{\psi}(C_1 - \theta C_2) \\ \frac{\phi_A}{\phi}(C_2 - \theta C_1) \end{bmatrix}.$$

Both elements of $D_\tau \mathbf{f}$ are positive, since $\psi_\tau < 0$ and $\phi_\tau < 0$. It follows that the elements of $D_\tau \mathbf{C}$ are negative, that is, both C_1 and C_2 are decreasing in τ. Both elements of $D_A \mathbf{f}$ negative, since $\psi_A > 0$ and $\phi_A > 0$. It follows that the elements of $D_A \mathbf{C}$ are positive, that is, both C_1 and C_2 are increasing in A.

For the second case of (21), consider the system defining this destructive equilibrium:

$$\begin{cases} \tilde{f}_1 \equiv C_1 + \theta C_2 - (1+\tau)AK_1/\tau = 0; \\ f_2 \equiv C_2 - \theta C_1 - \phi[(C_1 + \theta C_2)/C_2]^{1/\sigma} = 0. \end{cases}$$

Using the equilibrium conditions, it can be shown that

$$[D_\mathbf{C}\tilde{\mathbf{f}}]^{-1} = \frac{1}{\Delta_2} \begin{bmatrix} 1 + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{C_1}{C_2} & -\theta \\ \theta + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} & 1 \end{bmatrix},$$

where $\mathbf{C}$ is the vector of consumption levels, $\tilde{\mathbf{f}}$ is the vector of $\tilde{f}_1$ and f_2, and $\Delta_2 \equiv \det(D_\mathbf{C}\tilde{\mathbf{f}}) = 1 + \theta^2 + (C_2 - \theta C_1)/(\sigma C_2) > 0$.

For the effects of λ, note that, by the implicit function theorem, $D_\lambda \mathbf{C} = -[D_\mathbf{C}\tilde{\mathbf{f}}]^{-1} \cdot D_\lambda \tilde{\mathbf{f}}$ and

$$D_\lambda \tilde{\mathbf{f}} = \begin{bmatrix} -\frac{1}{\lambda}(C_1 + \theta C_2) \\ \frac{1}{(1-\lambda)\sigma}(C_2 - \theta C_1) \end{bmatrix}.$$

It follows immediately that $\partial C_1/\partial\lambda > 0$. Given that $\lambda < 1/2$, the sign of $\partial C_2/\partial\lambda$ coincides with

$$\frac{\theta}{\lambda} \cdot (C_1 + \theta C_2) + \frac{1}{\sigma} \cdot (C_2 - \theta C_1) \cdot \left(\frac{1}{\lambda} - \frac{1}{1-\lambda}\right) > 0.$$

For the effects of θ, note that $D_\theta \mathbf{C} = -[D_\mathbf{C}\tilde{\mathbf{f}}]^{-1} \cdot D_\theta \tilde{\mathbf{f}}$ and

$$D_\theta \tilde{\mathbf{f}} = \begin{bmatrix} C_2 \\ -C_1 + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{(1-\theta^2)C_1 - 2\theta^3 C_2}{\theta(1+\theta^2)} \end{bmatrix}.$$

It follows that the sign of $\partial C_1/\partial\theta$ coincides with that of

$$-\theta C_1 - C_2 - \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{2\theta^2(1 + \theta C_2)}{1 + \theta^2} < 0.$$

Next, although the sign of $\partial C_2/\partial\theta$ is ambiguous, $\partial C/\partial\theta < 0$, since

$$C_1 - C_2 - \theta(C_1 + C_2) - \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \left(1 + \frac{2\theta^3 + 2\theta^4 C_2 + (1-\theta^2)C_1 - 2\theta^3 C_2}{\theta(1+\theta^2)}\right) < 0,$$

where $C_1 < C_2$ in equilibrium and $2\theta^4 C_2 + (1-\theta^2)C_1 - 2\theta^3 C_2 > \theta C_2(1 - 3\theta^2 + 2\theta^3) > 0$, as $C_1 > \theta C_2$ and $\theta \in (0,1)$.

For the effects of τ, note that $D_\tau \mathbf{C} = -[D_{\mathbf{C}}\tilde{\mathbf{f}}]^{-1} \cdot D_\tau \tilde{\mathbf{f}}$ and

$$D_\tau \tilde{\mathbf{f}} = \frac{1}{1+\tau} \cdot \begin{bmatrix} \frac{1}{\tau}(C_1 + \theta C_2) \\ \frac{1}{\sigma}(C_2 - \theta C_1) \end{bmatrix}.$$

This immediately implies that $\partial C_2/\partial \tau < 0$. Although the sign of $\partial C_1/\partial \tau$ is ambiguous, the sign of $\partial C/\partial \tau$ coincides with that of

$$-\frac{C_1 + \theta C_2}{\tau(1+\tau)} \cdot \left[1 + \theta + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \left(\frac{C_1}{C_2} + 1\right)\right] - \frac{1-\theta}{\sigma} \cdot \frac{C_2 - \theta C_1}{1+\tau} < 0.$$

Finally, for the effects of A, note that $D_A \mathbf{C} = -[D_{\mathbf{C}}\tilde{\mathbf{f}}]^{-1} \cdot D_A \tilde{\mathbf{f}}$ and

$$D_A \tilde{\mathbf{f}} = -\frac{1}{A} \cdot \begin{bmatrix} C_1 + \theta C_2 \\ \frac{1}{\sigma}(C_2 - \theta C_1) \end{bmatrix}.$$

It follows immediately that $\partial C_2/\partial A > 0$. The sign of $\partial C_1/\partial A$ coincides with that of

$$\frac{1}{A} \cdot \left[C_1 + \theta C_2 + \frac{C_2 - \theta C_1}{\sigma} \cdot \left(\frac{C_1}{C_2} - \theta\right)\right] > 0.$$

For the thirds case of (21), the equilibrium individual and total consumption levels are given by

$$C_1 = A \cdot \left(\frac{1+\tau}{\tau} \cdot K_1 - \sqrt{\frac{\theta}{\tau} \cdot K_1 K_2}\right), \quad C_2 = A \cdot \sqrt{\frac{K_1 K_2}{\tau\theta}}, \quad C = A \cdot \left(\frac{1+\tau}{\tau} \cdot K_1 + (1-\theta) \cdot \sqrt{\frac{K_1 K_2}{\tau\theta}}\right).$$

The positive effect of A and the negative effect of θ on both C_1 and C_2 are obvious. While τ negatively affects both C_2 and C, its effect on C_1 is ambiguous. Specifically, $\partial C_1/\partial \tau < 0$ iff $k > \tau\theta/4$. Similarly, higher λ positively affects both C_2 and C, but its effect on C_1 is ambiguous.

Proof of Proposition 3. The steady-state levels of A follow immediately from (22). The economy gets stuck in the fear region iff $A_1^* < \hat{A}$ which yields

$$\frac{\alpha}{1-\alpha} < \frac{\sigma}{\sigma-1} \cdot \ln\left(\frac{\tau\theta}{1-\tau\theta^2} \cdot \frac{K_2^{1/\sigma}}{K_1}\right) \cdot \frac{1}{\ln((1+1/\tau\theta)K_1)}$$

defining the $\hat{\alpha}$ threshold. It is straightforward to show that the right-hand side of this inequality is increasing in τ and θ and decreasing in λ, that is, $\hat{\alpha}_\theta > 0$, $\hat{\alpha}_\lambda < 0$, $\hat{\alpha}_\tau > 0$.

If $A_1^* > \hat{A}$, then the economy is stuck in the KUJ-type region iff $A_2^* < \tilde{A}$ which yields

$$\frac{\alpha}{1-\alpha} < \frac{\sigma}{\sigma-1} \cdot \ln\left(\frac{K_1^{1/\sigma} + \theta K_2^{1/\sigma}}{(1-\theta^2)K_1}\right) \cdot \left(\ln\left(\frac{(1+\theta)K_1(K_1^{1/\sigma} + K_2^{1/\sigma})}{K_1^{1/\sigma} + \theta K_2^{1/\sigma}}\right)\right)^{-1}$$

defining the $\tilde{\alpha}$ threshold. First, note that the right-hand side of the above inequality is independent of τ, that is, $\tilde{\alpha}_\tau = 0$. Next, for the effect of θ, note that the numerator of the right-hand side is increasing in θ while the denominator is decreasing in θ, since the sign of the respective derivative coincides with that of $K_1^{1/\sigma} - K_2^{1/\sigma} < 0$. Hence, $\tilde{\alpha}_\theta > 0$. For the effect of λ, note that the numerator of the right-hand side is decreasing in λ, while the denominator is increasing in λ since the sign of the respective derivative coincides with that of

$$\left(\frac{\lambda}{1-\lambda}\right)^{\frac{1}{\sigma}} - \frac{1-\sigma\theta}{\sigma} \cdot \frac{\lambda}{1-\lambda} + \frac{\sigma-1}{\sigma} \cdot \left(1 + \frac{\theta}{\lambda}\left(\frac{1-\lambda}{\lambda}\right)^{\frac{1-\sigma}{\sigma}}\right) + \theta \cdot \left(\frac{\sigma+1}{\sigma} + \left(\frac{1-\lambda}{\lambda}\right)^{\frac{1}{\sigma}}\right) > 0.$$

The latter holds because the sum of the first two terms is always positive, as $\lambda < 1/2$ and $\sigma > 1$. Hence, $\tilde{\alpha}_\lambda < 0$.

Proof of Proposition 4. As follows from (2), (5), and (19), utilities in the long-run FE are given by

$$U_1^{\mathrm{F}} = \left(\frac{1}{1-\sigma} - \frac{\tau}{\tau-1}\right)\cdot\left(A^{\mathrm{F}}K_1\right)^{\frac{1-\sigma}{\sigma}},$$
$$U_2^{\mathrm{F}} = \left(\frac{1}{1-\sigma}\cdot\left[\frac{1-\tau\theta^2}{\theta(\tau-1)}\right]^{1-\sigma} - \frac{1}{\theta(\tau-1)}\cdot\frac{K_1}{K_2}\right)\cdot\left(A^{\mathrm{F}}K_1\right)^{\frac{1-\sigma}{\sigma}},$$

where A^{F} is defined by (24). In the KUJE, as follows from (16), utilities are given by

$$U_i^{\mathrm{KUJ}} = \left(K_i^{\frac{1-\sigma}{\sigma}}\cdot\left[\frac{1}{1-\sigma} - \frac{1}{1-\theta^2}\right] - \frac{\theta}{1-\theta^2}\cdot\frac{K_j^{\frac{1}{\sigma}}}{K_i}\right)\cdot\left(A^{\mathrm{KUJ}}\right)^{\frac{1-\sigma}{\sigma}}, \quad i,j=1,2,\ i\neq j,$$

where A^{KUJ} is defined by (25). For Agent 1, $U_1^{\mathrm{KUJ}} > U_1^{\mathrm{F}}$ iff

$$\left(\frac{1}{\sigma-1} + \frac{1}{1-\theta^2} + \frac{\theta}{1-\theta^2}\cdot k^{-\frac{1}{\sigma}}\right)\cdot\left(\frac{A^{\mathrm{F}}}{A^{\mathrm{KUJ}}}\right)^{\frac{\sigma-1}{\sigma}} < \frac{1}{\sigma-1} + \frac{\tau}{\tau-1}.$$

Using (24), (25), and the substitution of variable $\mathcal{K} \equiv 1 + k^{-\frac{1}{\sigma}} \in [\widetilde{\mathcal{K}}, \widehat{\mathcal{K}}]$, where $\widetilde{\mathcal{K}} \equiv 1 + \tilde{k}^{-\frac{1}{\sigma}}$ and $\widehat{\mathcal{K}} \equiv 1 + \hat{k}^{-\frac{1}{\sigma}}$, the above inequality can be transformed into

$$g_1(\mathcal{K}) \equiv \left(\frac{1}{\sigma-1} + \frac{1}{1+\theta} + \frac{\theta}{1-\theta^2}\cdot\mathcal{K}\right) - \left(\frac{1}{\sigma-1} + \frac{\tau}{\tau-1}\right)\cdot\left[\frac{\theta(\tau-1)}{(1+\tau\theta)(1-\theta)}\cdot\mathcal{K}\right]^{\frac{\alpha(\sigma-1)}{\sigma-\alpha}} < 0,$$

where $g_1(\widetilde{\mathcal{K}}) = 0$ and $g_1(\mathcal{K})$ is strictly convex. Furthermore, direct computation shows that $g_1'(\widetilde{\mathcal{K}}) > 0$ iff

$$\alpha < \alpha_1 \equiv \frac{\sigma(1+\tau\theta)}{\sigma\tau(1+\theta) + \theta(\tau-1)}.$$

Hence, for all $\alpha \leqslant \alpha_1$, $g_1(\mathcal{K}) > 0$ on the segment $(\widetilde{\mathcal{K}}, \widehat{\mathcal{K}}]$, that is, $U_1^{\mathrm{KUJ}} < U_1^{\mathrm{F}}$, while for $\alpha > \alpha_1$, $g_1(\mathcal{K})$ has at most one root on that segment, that is, $\exists!\ \bar{k}_2 \in [\hat{k}, \tilde{k})$ such that $U_1^{\mathrm{KUJ}} > U_1^{\mathrm{F}}$ for all $k \in (\bar{k}_2, \tilde{k}]$. Note also that in the short-run (for the current generation) the comparison between utilities is equivalent to setting $\alpha = 0$ in the above calculation, since productivity changes with a lag and only affects future generations. Hence, in the short run $g_1(\mathcal{K})$ is always positive on $(\widetilde{\mathcal{K}}, \widehat{\mathcal{K}}]$ and Agent 1 is always better off staying in the long-run fear equilibrium.

For Agent 2, $U_2^{\mathrm{KUJ}} > U_2^{\mathrm{F}}$ iff

$$\left(k^{-\frac{1}{\sigma}}\cdot\left[\frac{1}{\sigma-1} + \frac{1}{1-\theta^2}\right] + \frac{\theta}{1-\theta^2}\right)\cdot\left(\frac{A^{\mathrm{F}}}{A^{\mathrm{KUJ}}}\right)^{\frac{\sigma-1}{\sigma}} < \frac{1}{\sigma-1}\cdot\tilde{k}^{\frac{\sigma-1}{\sigma}}\cdot k^{-\frac{1}{\sigma}} + \frac{1}{\theta(\tau-1)}.$$

Using the same substitution as before, this inequality can be rewritten as

$$g_2(\mathcal{K}) \equiv \frac{\tilde{k}^{\frac{\sigma-1}{\sigma}}\cdot(\mathcal{K}-1)^{\sigma}}{\sigma-1} - \left[\frac{\theta(\tau-1)\cdot\mathcal{K}}{(1+\tau\theta)(1-\theta)}\right]^{\frac{\alpha(1-\sigma)}{\sigma-\alpha}}\cdot\left[\frac{\mathcal{K}}{\sigma-1} + \frac{\mathcal{K}}{1-\theta^2} - \frac{\sigma+\theta}{(1+\theta)(\sigma-1)}\right] + \frac{1}{\theta(\tau-1)} > 0,$$

where $g_2(\widetilde{\mathcal{K}}) = 0$ and $g_2(\mathcal{K})$ is strictly convex. Furthermore, direct computation shows that $g_2'(\widetilde{\mathcal{K}}) > 0$ iff

$$\alpha > \alpha_2 \equiv \frac{\sigma\theta^2(1+\tau\theta)}{\sigma(1+\theta) - \theta^2(\tau-1)}.$$

Hence, for all $\alpha > \alpha_2$, $g_2(\mathcal{K}) > 0$ on the segment $(\widetilde{\mathcal{K}}, \widehat{\mathcal{K}}]$, that is, $U_2^{\text{KUJ}} > U_2^{\text{F}}$, while for $\alpha < \alpha_2$, $g_2(\mathcal{K})$ has at most one root on that segment. Since $\theta \in (0,1)$ and $\tau\theta < 1$, we also know that $\alpha_1 > \alpha_2$ which leads to the final statement of the long-run part of the proposition, where $\bar{\alpha} \equiv \alpha_1$. To complete the statement of the short-run part of the proposition, note that, for $\alpha = 0$, $g_2'(\widetilde{\mathcal{K}}) < 0$ and $g_2(\widehat{\mathcal{K}}) < 0$ iff

$$\left(\frac{1}{1-\theta^2} + \frac{1}{\sigma - 1}\right) \cdot \left(\frac{1+\theta^2}{2}\right)^{\frac{\sigma-1}{\sigma}} - \frac{1}{\sigma-1} - \frac{1+\theta^2}{2(1-\theta^2)} > 0.$$

Let $q \equiv (1+\theta^2)/2 \in (1/2, 1)$ and $s \equiv (\sigma-1)/\sigma \in (0,1)$. After re-arranging, the above inequality becomes

$$\vartheta(q) \equiv [s + 2(1-s)(1-q)]q^s + (2-3s)q - 2(1-s) > 0.$$

Note that: 1) $\vartheta(1/2) = (1/2)^s + s/2 - 1 < 0$ since it is strictly convex in s on the $(0,1)$ interval and is equal to zero at its endpoints, 2) $\vartheta(1) = 0$, 3) $\vartheta'(1) = s(s-1) < 0$, and 4) $\vartheta''(q) < 0$. Hence $\exists!\ \bar{q} \in (1/2, 1)$, such that $\vartheta(q) > 0$ iff $q > \bar{q}$. This implies that $\exists!\ \bar{\theta} \in (0,1)$, such that $g_2(\widehat{\mathcal{K}}) < 0$ (and thus, $U_2^{\text{KUJ}} < U_2^{\text{F}}$ in the short run) $\forall k \in (\hat{k}, \tilde{k})$ iff $\theta > \bar{\theta}$. If $\theta < \bar{\theta}$, $\exists!\ \bar{k}_1 \in (\hat{k}, \tilde{k})$, such that $U_2^{\text{KUJ}} < U_2^{\text{F}}$ in the short run iff $k > \bar{k}_1$. This, along with the fact that Agent 1 is always better off in the FE under $\alpha = 0$, completes the proof of the short-run part of the proposition.

Proof of Proposition 5. First, note that both U_1^{F} and U_1^{KUJ} are strictly increasing in k for Agent 1, that is, he is always supportive of redistribution. The critical role is thus played by Agent 2. To examine U_2^{F} and U_2^{KUJ} as functions of k rewrite the equilibrium utilities as

$$U_2^{\text{F}}(k) = \left(\frac{1}{1-\sigma} \cdot \tilde{k}^{\frac{\sigma-1}{\sigma}} - \frac{1}{\theta(\tau-1)} \cdot k\right) \cdot \left(\frac{k}{1+k} \cdot K\right)^{\frac{1-\sigma}{\sigma-\alpha}} \cdot \left[\frac{1+\tau\theta}{\theta(\tau-1)}\right]^{\frac{\alpha(1-\sigma)}{\sigma(\sigma-\alpha)}},$$

$$U_2^{\text{KUJ}}(k) = \left(\frac{1}{1-\sigma} - \frac{1}{1-\theta^2} - \frac{\theta}{1-\theta^2} \cdot k^{\frac{1}{\sigma}}\right) \cdot \left(\frac{1}{1+k} \cdot K\right)^{\frac{1-\sigma}{\sigma-\alpha}} \cdot \left(\frac{k^{\frac{1}{\sigma}}}{1-\theta}\right)^{\frac{\alpha(1-\sigma)}{\sigma-\alpha}}.$$

Consider first the behavior of $U_2^{\text{F}}(k)$. Direct calculations show that $dU_2^{\text{F}}(k)/dk < 0$ iff

$$h_1(k) \equiv -(\sigma-\alpha)k^2 - (1-\alpha)k + \delta < 0, \quad \delta \equiv \theta(\tau-1)\tilde{k}^{\frac{\sigma-1}{\sigma}}.$$

The short-run part of the proposition corresponds to the case $\alpha = 0$. In this case, in particular, it is straightforward to show that $h_1(\tilde{k}) < 0$, that is, the maximum value of $h_1(k)$ on $[\hat{k}, \tilde{k}]$ is strictly less that $\tilde{k}$. Furthermore, if $h_1(\hat{k}) > 0$, $h_1(k) < 0$ (hence, $dU_2^{\text{F}}(k)/dk < 0$) $\forall k \in [\hat{k}, \tilde{k}]$, in which case redistribution to Agent 1 is never optimal in the fear equilibrium from the viewpoint of Agent 2 (that is, Pareto improving redistribution towards KUJE is not feasible). If $h_1(\hat{k}) > 0$, $h_1(k)$ has one positive root $k^* = \sqrt{1+4\sigma\delta}/(2\sigma)$ falling in the interval $[\hat{k}, \tilde{k}]$ which is the global maximum of $U_2^{\text{F}}(k)$ on that interval. Note that in this latter case redistribution to KUJE might be Pareto improving for some range of initial k (incidentally, $U_2^{\text{KUJ}}(k)$ is strictly decreasing in k in the short-run case $\alpha = 0$). Specifically, such range will be non-empty iff $U_2^{\text{F}}(\hat{k}) < U_2^{\text{F}}(\tilde{k})$. This inequality is equivalent to

$$\left(\frac{1+\theta^2}{2} \cdot \frac{1+\tilde{k}}{1+\hat{k}}\right)^{\frac{\sigma-1}{\sigma}} < \frac{\sigma+1+\theta^2(\sigma-1-2\tau)}{2(\sigma-\tau\theta^2)}. \tag{A.12}$$

Denote the left-hand side of this expression as $\mathcal{L}_1(\theta)$ and the right-hand side of it as $\mathcal{R}_1(\theta)$. First note that $\mathcal{L}(0) - \mathcal{R}_1(0) = (1/2)^{(\sigma-1)/\sigma} - 1/(2\sigma) - 1/2 < 0$ since $\sigma > 1$. In addition,

$$\mathcal{L}_1(1/\tau) - \mathcal{R}_1(1/\tau) = \left[\frac{2(1+\tau^2)}{1+3\tau^2}\right]^{\frac{\sigma-1}{\sigma}} - \frac{2\tau(\tau-1) + (\sigma-1)(1+\tau^2)}{2\tau(\sigma\tau-1)},$$

which is a strictly increasing function of σ with $\inf\{\mathcal{L}_1(1/\tau) - \mathcal{R}_1(1/\tau)\} = 0$. Furthermore, simple differentiation shows that $\mathcal{R}_1(\theta)$ is strictly increasing and convex if and only if $\sigma > \tau$, and it is strictly decreasing and concave if and only if $\sigma < \tau$. The left-hand side $\mathcal{L}_1(\theta)$ is strictly increasing. Hence, in the case when $\sigma < \tau$ there is a unique $\bar{\bar{\theta}}$ such that $\mathcal{L}_1(\bar{\bar{\theta}}) = \mathcal{R}_1(\bar{\bar{\theta}})$. For the case $\sigma > \tau$, rewrite (A.12) as

$$\frac{1+\theta^2}{2} \cdot (1+\tilde{k}) < \mathcal{R}_1(\theta)^{\frac{\sigma}{\sigma-1}} \cdot (1+\hat{k}).$$

Both sides of this inequality are strictly increasing and convex in θ as products of strictly increasing positive convex functions. Given that $\mathcal{L}_1(0) < \mathcal{R}_1(0)$ and $\mathcal{L}_1(1/\tau) > \mathcal{R}_1(1/\tau)$, this implies a single intersection, as in the case with $\sigma < \tau$. Putting together the two cases yields the short-run part of the proposition.

Now examine the long-run equilibrium value of $U_2^{\mathrm{KUJ}}(k)$. Direct calculations show that the sign of $dU_2^{\mathrm{KUJ}}(k)/dk$ coincides with that of $h_2(k) \equiv \mathcal{L}_2(k) - \mathcal{R}_2(k)$, where

$$\mathcal{L}_2(k) \equiv \rho_2 \cdot \frac{\rho_3 - \rho_4 k^{\frac{1}{\sigma}}}{1 + k^{\frac{1}{\sigma}}}, \quad \mathcal{R}_2(k) \equiv \sigma\rho_1 \cdot \frac{\rho_3 k^{\frac{\sigma-1}{\sigma}} - \rho_4 k}{1+k} + \rho_4,$$

and

$$\rho_1 \equiv \frac{1-\sigma}{\sigma-\alpha} < 0, \quad \rho_2 \equiv \frac{\alpha(1-\sigma)}{\sigma-\alpha} < 0, \quad \rho_3 \equiv \frac{1}{1-\sigma} - \frac{1}{1-\theta^2} < 0, \quad \rho_4 \equiv \frac{\theta}{1-\theta^2} > 0.$$

Direct calculation also shows that $h_2(1) = -(\sigma-\theta)/[2(1-\theta)] - \theta/(1-\theta^2) < 0$, that is, redistribution up to perfect equality ($k = 1$) is never optimal for Agent 2. Next,

$$\frac{d\mathcal{L}_2(k)}{dk} = -\frac{\rho_2(\rho_3+\rho_4)}{\sigma} \cdot \frac{k^{\frac{1-\sigma}{\sigma}}}{(1+k^{\frac{1}{\sigma}})^2} < 0,$$

since $\rho_2 < 0$ and $\rho_3 + \rho_4 < 0$. Furthermore, $d^2\mathcal{L}_2(k)/dk^2 > 0$, that is, $\mathcal{L}_2(k)$ is a strictly decreasing convex function of k. On the other hand,

$$\frac{d\mathcal{R}_2(k)}{dk} = \rho_1 \cdot \frac{\rho_3(\sigma-1)k^{-\frac{1}{\sigma}} - \rho_3 k^{\frac{\sigma-1}{\sigma}} - \sigma\rho_4}{(1+k)^2} = 0 \quad \Longleftrightarrow \quad k^{\frac{1}{\sigma}} = \frac{\rho_3}{\sigma\rho_4} \cdot (\sigma - 1 - k).$$

The latter equation can have at most one root on the interval $[\tilde{k}, 1)$. Hence, $\mathcal{R}_2(k)$ is either strictly increasing or has a unique interior global maximum on $[\tilde{k}, 1)$. It follows that $\mathcal{L}_2(k)$ and $\mathcal{R}_2(k)$ have a unique intersection on $[\tilde{k}, 1)$ iff $\mathcal{L}_2(\tilde{k}) > \mathcal{R}_2(\tilde{k})$. The latter holds iff

$$\alpha > \bar{\bar{\alpha}} \equiv \sigma \cdot \frac{\theta(1+\sigma\tilde{k}) + (\sigma-\theta^2)\tilde{k}^{\frac{\sigma-1}{\sigma}}}{\theta(1+\sigma\tilde{k}^{\frac{1}{\sigma}}) + \sigma - \theta^2} \cdot \frac{1+\tilde{k}^{\frac{1}{\sigma}}}{1+\tilde{k}}.$$

Note that $\bar{\bar{\alpha}} = 0$ for $\theta = 0$ and $\bar{\bar{\alpha}} = \sigma > 1$ for $\theta = 1/\tau$. Hence, by continuity, there is a region of small enough values of θ such that $\bar{\bar{\alpha}} < 1$. Similarly, $\bar{\bar{\alpha}} = \sigma\theta/(\sigma+\theta-\theta^2) < 1$ for $\tau = 1$ and $\bar{\bar{\alpha}} = \sigma > 1$ for $\tau = 1/\theta$. Hence, by continuity, there is a region of small enough values of τ such that $\bar{\bar{\alpha}} < 1$. This concludes the proof of the long-run part of proposition 5.

Supplementary Appendix B

"The Two Sides of Envy"

Journal of Economic Growth

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This appendix adds bequest dynamics to the basic model from "The Two Sides of Envy."

B Envy, inequality, and bequest dynamics

We introduce bequests in the basic model of section 3 in order to explore the role of inequality dynamics in driving the endogenous evolution of the economy through alternative envy regimes. We make two simplifying assumptions to focus on bequest dynamics and reduce the complexity associated with a wide range of qualitatively similar equilibria. First, assume that the productivity parameter is constant and equal to 1. Second, assume that in the investment stage of the game the amount of exerted effort L_i, $i = 1, 2$, is not bounded from above. The latter assumption effectively makes part 1 of proposition 1 the only relevant case.

As before, initial population consists of 2 homogeneous groups (or their representative agents), the poor and the rich, with initial endowments K_{10} and $K_{20} > K_{10}$. Each person has 1 child, so that population is constant and represents the descendants of the two original groups. Parents care about their children and leave bequests, b_{it}, $i = 1, 2$, at the end of each period t.[1] In particular, they derive utility not just from relative consumption but the relative Cobb-Douglas aggregator of consumption, c_{it}, and bequest, b_{it}:

$$U_{it} = \frac{\pi(c_{it}^{1-\eta} b_{it}^{\eta} - \theta c_{jt}^{1-\eta} b_{jt}^{\eta})^{1-\sigma}}{1-\sigma} - L_{it}, \tag{B.1}$$

where $i, j = 1, 2$, $i \neq j$, $0 < \eta < 1$ parameterizes the fraction of final output allocated to bequest, and $\pi \equiv [(1-\eta)^{1-\eta}\eta^{\eta}]^{\sigma-1}$ is a normalization constant. Final output, C_{it}, is

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[1] These bequests may represent any kind of investment in children that increases the productivity of their effort, for example, expenditure on human capital. Matt (2003) provides anecdotal evidence on how spending on children is itself an object of rivalry between parents, supporting the type of preferences put forward in equation (B.1).

split between consumption and bequest, that is, $b_{it} + c_{it} = C_{it}$. This formulation leaves the workings of the basic model from section 3 intact while introducing dynamic linkages.

The initial endowment of generation $t+1$, K_{it+1}, is assumed to depend on the parents' endowment and their investment in children:

$$K_{it+1} = K_{it}^{\beta} b_{it} = K_{it}^{\beta} \eta C_{it}, \tag{B.2}$$

where $i = 1, 2$, and $0 < 1 - \beta < 1$ is the rate of geometric depreciation of parental endowment.[2] Note that part 1 of proposition 1 holds in each period, and the level of initial inequality in period $t+1$ is determined endogenously by the difference equation

$$k_{t+1} \equiv \frac{K_{1t+1}}{K_{2t+1}} = k_t^{\beta} \cdot \frac{C_{1t}}{C_{2t}}, \tag{B.3}$$

given the initial condition $k_0 \in (0, 1)$. The joint dynamics of K_{1t} and K_{2t} depends on the type of equilibrium, in which the economy resides, and the equilibrium next period is, in turn, determined by the economic outcomes of the current period. Lemma B.1 characterizes this dynamical system and follows directly from proposition 1 and the law of motion (B.2).

Lemma B.1. (Dynamics of endowments). The two-dimensional dynamical system for K_{1t} and K_{2t} is given by

$$\begin{bmatrix} K_{1t+1} \\ K_{2t+1} \end{bmatrix} = \begin{cases} \frac{\eta}{1-\theta^2}[K_{1t}^{\beta}(K_{1t}^{1/\sigma} + \theta K_{2t}^{1/\sigma}), K_{2t}^{\beta}(K_{2t}^{1/\sigma} + \theta K_{1t}^{1/\sigma})], & \text{if} \quad k_t \geqslant \tilde{k}; \\ \frac{\eta}{\theta(\tau-1)}[\tau\theta K_{1t}^{1/\sigma+\beta}, K_{2t}^{\beta} K_{1t}^{1/\sigma}], & \text{if} \quad \hat{k} \leqslant k_t < \tilde{k}; \\ \eta[K_{1t}^{\beta} \cdot C_1^*(K_{1t}, K_{2t}), K_{2t}^{\beta} \cdot C_2^*(K_{1t}, K_{2t})], & \text{if} \quad k_t < \hat{k}, \end{cases} \tag{B.4}$$

where $C_1^*(K_{1t}, K_{2t})$ and $C_2^*(K_{1t}, K_{2t})$ are the final outputs in the DE corresponding to the first case of (21) and the thresholds $\tilde{k}$ and $\hat{k}$ are defined in (28).

The thresholds $\tilde{k}$ and $\hat{k}$ divide the (K_{1t}, K_{2t}) phase plane into three regions according to the types of equilibria (see figures B1 and B2): KUJE, FE, and DE. In each of these regions the motion is governed by the corresponding part of the dynamical system (B.4). To rule out explosive dynamics it is assumed throughout this section that $\sigma(1-\beta) > 1$ which also implies that the results of all previous sections hold, since $\sigma > 1$.

It is useful to examine the companion one-dimensional difference equation (B.3) driving the dynamics of inequality. Its properties are established in the following lemma.

[2] Persistence of endowments is introduced to make the dynamics more realistic. If $\beta = 0$, the qualitative results remain unchanged, but the economy will not stay in the fear equilibrium for more than one period.

Lemma B.2. (Dynamics of inequality). The dynamics of k_t is given by

$$k_{t+1} = \begin{cases} g_1(k_t) \equiv [k_t^{1/\sigma+\beta} + \theta k_t^{\beta}]/[1+\theta k_t^{1/\sigma}], & \text{if} \quad k_t \geqslant \tilde{k}; \\ g_2(k_t) \equiv \tau\theta k_t^{\beta}, & \text{if} \quad \hat{k} \leqslant k_t < \tilde{k}; \\ g_3(k_t), & \text{if} \quad k_t < \hat{k}, \end{cases} \tag{B.5}$$

where $g_3(k_t)$ is implicitly given by

$$k_t = \frac{\tau(1+\theta^2)}{2\theta(1+\tau)^2} \cdot \frac{(z_{t+1}+\theta)^2}{z_{t+1}} \cdot \left(\frac{z_{t+1}-\theta}{1-\theta z_{t+1}}\right)^{\sigma}, \quad z_{t+1} \equiv k_{t+1}/k_t^{\beta}. \tag{B.6}$$

Moreover, $k_{t+1} > k_t$ for all $k_t \in (0,1)$.

Further analysis focuses on the dynamics of the system in the fear region and the KUJ region. The two lemmas below provide its characterization.

Lemma B.3. (KUJ region dynamics). The system converges to a unique stable "equal" long-run steady state $\bar{K}_1^{\text{KUJ}} = \bar{K}_2^{\text{KUJ}} = \bar{K} = [\eta/(1-\theta)]^{\frac{\sigma}{\sigma(1-\beta)-1}}$. The steady-state levels of output are equal to $\bar{Y}_1^{\text{KUJ}} = \bar{Y}_2^{\text{KUJ}} = \bar{Y} = \bar{K}^{1/\sigma}/(1-\theta)$. The evolution of endowments is determined by the loci

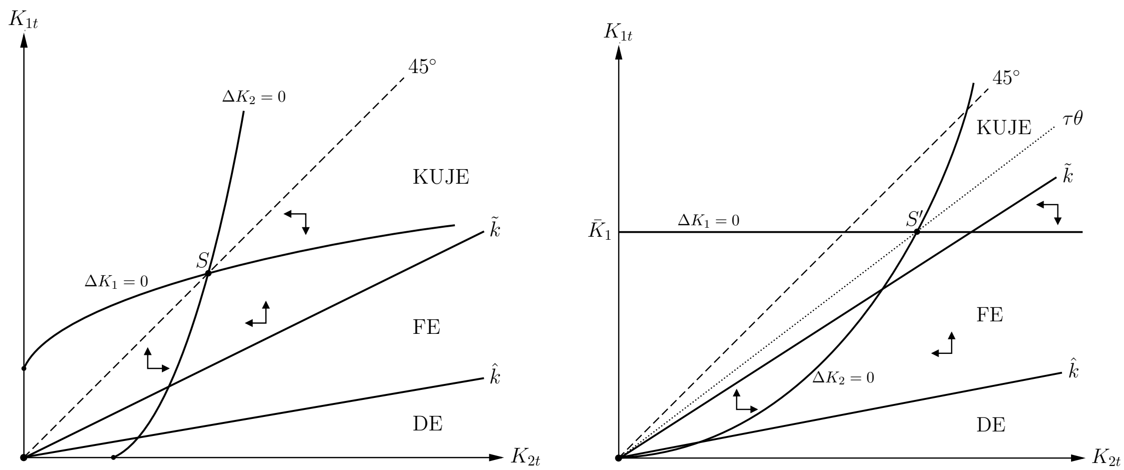
$$\Delta K_i \equiv K_{it+1} - K_{it} = 0: \quad K_{it}^{1/\sigma} + \theta K_{jt}^{1/\sigma} = K_{it}^{1-\beta} \cdot (1-\theta^2)/\eta, \quad i,j = 1,2, \quad i \neq j,$$

where $dK_{it}/dK_{jt} > 0$ and $d^2K_{it}/dK_{jt}^2 < 0$.

Figure B1(a) shows the dynamics in the KUJ region. The descendants of the initially poor eventually catch up with the rich dynasty, and in the steady state both have the same endowments and consumption levels.

Lemma B.4. (Fear region dynamics). In the fear region the system moves towards the unique stable "unequal" long-run steady state $\bar{K}_1 = [\eta\tau/(\tau-1)]^{\frac{\sigma}{\sigma(1-\beta)-1}}$, $\bar{K}_2 = \bar{K}_1 \cdot (\tau\theta)^{\frac{1}{\beta-1}}$. The corresponding levels of output are $\bar{Y}_1 = [\tau/(\tau-1)]\bar{K}_1^{1/\sigma}$ and $\bar{Y}_2 = \bar{K}_1^{1/\sigma}/[\theta(\tau-1)]$. This steady state is, however, unattainable, since it is located in the KUJ region, that is, the system enters the KUJ region before reaching the fear steady state. The evolution of endowments is determined by the loci

$$\begin{aligned} \Delta K_1 = 0: &\quad K_{1t} = \bar{K}_1 = [\eta\tau/(\tau-1)]^{\frac{\sigma}{\sigma(1-\beta)-1}}; \\ \Delta K_2 = 0: &\quad K_{1t} = K_{2t}^{\sigma(1-\beta)} \cdot [\theta(\tau-1)/\eta]^{\sigma}. \end{aligned}$$



(a) The KUJ region (b) The fear of envy region

Figure B1: Dynamics of bequests and the steady states.

Figure B1(b) shows the dynamics in the fear region. It is instructive to look at the comparative statics of the long-run levels of output with respect to τ and θ. They resemble the results of the static model: in the KUJ steady state, outputs are increasing in θ and independent of τ; in the (unattainable) fear steady state, the output of group 1 is independent of θ and decreasing in τ, while the output of group 2 is decreasing in θ and τ. Note also that, despite their qualitatively different nature, as $\tau\theta \to 1$, the two long-run equilibria get closer and coincide in the limit.

Given lemmas B.2–B.4, it is easy to establish how the possible trajectories look. The long-run convergence result is stated in proposition B.1 and illustrated in figure B2.

Proposition B.1. (Long-run convergence). Given the initial conditions $\{K_{10}, K_{20}\}$, such that $k_0 \in (0, 1)$, the endowments converge to a unique stable long-run "equal" steady state of the KUJ region, $\bar{K}$. Inequality decreases monotonically along the transition path.

Thus, if the economy starts off, say, in the destructive region, it experiences a transition to the KUJ steady state, possibly passing through the fear region and staying there for a while. Initially, destructive envy and the fear of it reduce original inequality of endowments by discouraging productive effort of the rich or destroying part of their output. This leads to more equal investment opportunities for future generations who eventually find it optimal to compete productively.

The changes in tolerance parameters τ and θ can delay or accelerate the transition between alternative envy regimes. For instance, a decrease in θ due to the diffusion of

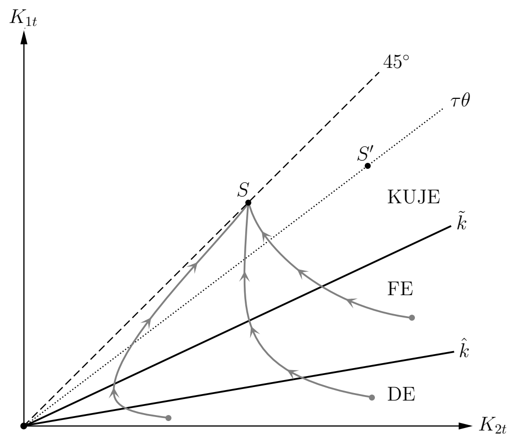
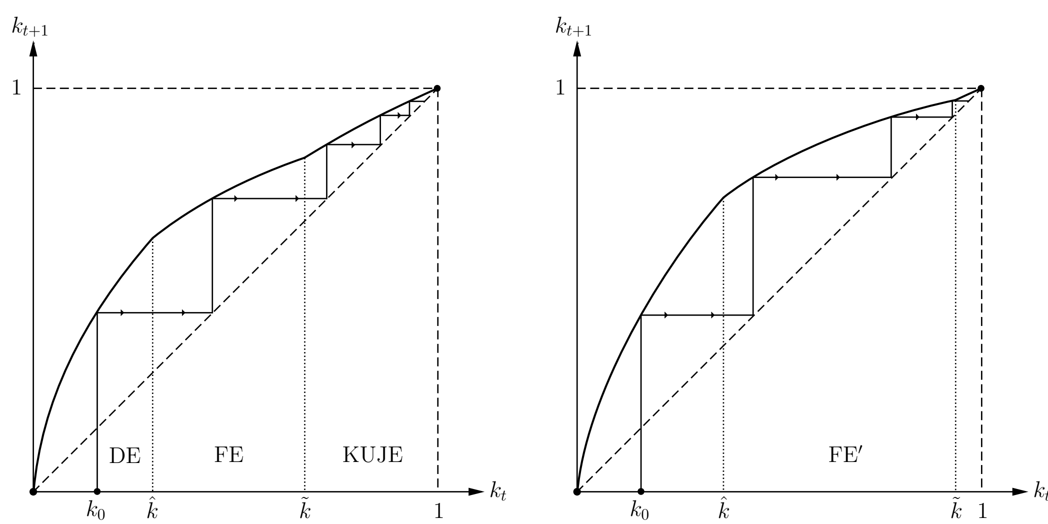


Figure B2: Evolution through envy regimes.

religious teachings condemning envy would lower the threshold $\tilde{k}$ contributing to a faster transition to the KUJ region. Similar to the discussion in section 4, once the society switches to a different equilibrium type, the effect of θ on economic performance changes its sign. With regard to τ, it is easy to show that deterioration of institutions (higher τ) has the following consequences: 1) the threshold $\tilde{k}$ increases; 2) the scope of the fear region $(\tilde{k} - \hat{k})$ extends; 3) the long-run fear steady state moves closer to the long-run KUJ steady state (see figure B3).



(a) Initial dynamics

(b) Dynamics for higher τ

Figure B3: Dynamics of inequality and property rights.

Proof of Lemma B.2. The functional forms of $g_i(k_t)$, $i = 1, 2$, follow directly from (16), (19), and (B.3). The form of $g_3(k_t)$ follows from (26), (27) and (B.3). It is straightforward to establish by differentiation that $g_1(k)$ and $g_2(k)$ are strictly increasing and concave, given that $\beta < 1$ and $\sigma(1-\beta) > 1$. Furthermore, 1) $g_1(1) = 1$, 2) $g_1(\tilde{k}) = g_2(\tilde{k}) > \tilde{k}$, and 3) $g_2(\hat{k}) = g_3(\hat{k}) > \hat{k}$. The second property holds iff $\tau\theta > \tilde{k}^{1-\beta} = [\theta(\tau-1)/(1-\tau\theta^2)]^{\sigma(1-\beta)}$ which is always true, since $\tau\theta > \theta(\tau-1)/(1-\tau\theta^2)$ and $\sigma(1-\beta) > 1$. The third property holds iff $\tau\theta > \hat{k}^{1-\beta}$ which is always true, as follows from property 2 above and the fact that $\hat{k} < \tilde{k}$. Together these properties imply that $k_{t+1} > k_t$ for $k_t \geqslant \hat{k}$.

Finally, if there is a steady state $\bar{k}$ in the destructive segment, then $g_3(\bar{k}) = \bar{k}$. Making a substitution $\kappa = \bar{k}^{1-\beta}$ and rearranging terms yields the equation defining the steady state:

$$\frac{\kappa^{2-\beta}}{(\kappa+\theta)^{2(1-\beta)}} = \chi\left(\frac{\kappa-\theta}{1-\theta\kappa}\right)^{\sigma(1-\beta)}, \tag{B.7}$$

where $\chi \equiv [\tau(1+\theta^2)/2\theta(1+\tau)^2]^{1-\beta}$ and in equilibrium $\kappa > \theta$. It is easy to show that $\mathcal{R}(\kappa)$, the right-hand side of (B.7), is strictly increasing and convex. Also, $\mathcal{L}(\kappa)$, the left-hand side of (B.7), is strictly increasing. To show that it is concave for $\kappa > \theta$, note that the sign of $\mathcal{L}''(\kappa)$ is determined by the sign of $\tilde{\mathcal{L}}(\kappa) \equiv (\kappa+\theta)(2-\beta)(\beta\kappa+\theta(1-\beta)) - \kappa(3-2\beta)(\beta\kappa+\theta(2-\beta))$. At $\kappa = \theta$ this expression is negative, since $\beta < 1$. Moreover, it is strictly decreasing for the same reason. Hence, $\mathcal{L}''(\kappa) < 0$ for $\kappa > \theta$. This implies that there exists at most one solution to (B.7). If there is no solution, the proof is finished. Assume now that $\bar{k}$ exists. In this case $\bar{k} > \hat{k}$. To see this, note first that $\hat{k} < (\tau\theta)^{1/(1-\beta)}$. Next, $\kappa > \tau\theta$ since $\mathcal{L}(\tau\theta) > \mathcal{R}(\tau\theta)$. Hence, $\bar{k} \equiv \kappa^{1/(1-\beta)} > (\tau\theta)^{1/(1-\beta)} > \hat{k}$, that is, $k_{t+1} > k_t$ for $k_t \in (0, \hat{k})$.

Proof of Lemma B.3. It follows from the properties of $g_1(k_t)$ that k_t monotonically converges to 1 in the KUJ region. The expression for $\Delta K_i = 0$ comes from (B.4) and may be rewritten as $K_{jt} = K_{it}[(\rho K_{it}^{1-\beta-1/\sigma} - 1)/\theta]^\sigma$, where $\rho \equiv (1-\theta^2)/\eta$. Differentiation yields $dK_{jt}/dK_{it} > 0$ and $d^2K_{jt}/dK_{it}^2 > 0$, since $\sigma(1-\beta) > 1$. This implies the stated properties of the $\Delta K_i = 0$ loci. The expression for $\bar{K}$ follows from solving $\Delta K_1 = \Delta K_2 = 0$ and (16) then gives $\bar{Y}$.

Proof of Lemma B.4. The equations for $\Delta K_i = 0$ come from (B.4). Since in the fear region $K_{1t+1} = \eta\tau K_{1t}^{1/\sigma+\beta}/(\tau-1)$ and $\sigma(1-\beta) > 1$, K_{1t} converges to $\bar{K}_1$. Plugging this in the $\Delta K_2 = 0$ equation yields $\bar{K}_2$. The output levels follow from (19). To see that the long-run fear equilibrium is in the KUJ region, note that the implied steady-state level of inequality is $(\tau\theta)^{1/(1-\beta)}$ which exceeds $\tilde{k} = [\theta(\tau-1)/(1-\tau\theta^2)]^\sigma$, since $\sigma(1-\beta) > 1$ and $\theta(\tau-1)/(1-\tau\theta^2) < \tau\theta < 1$.